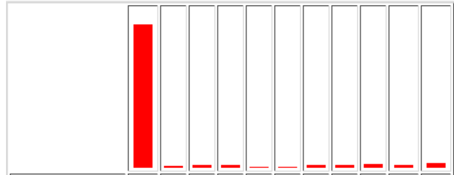
Information

Take care of **Practice Test 3** ASAP

17	18	19	20	21	22	23
Practice Test 3 is available	**EMCF23 due at 9am** Notes: **page**, **4-per** **See Wednesday's Video** Homework 8 due		**EMCF24 due at 9am** Notes: **page**, **4-per**, **video notes**, **video** **No Office Hours Today** **Homework 9 posted**		**EMCF25 due at 9am** Notes: **page**, **4-per**, **video notes**, **video** Quiz in lab/workshop	**Quiz 8 closes** (9.6-9.8) **2011 Test 3 Review: slides, video** **2012 Test 3 Review: slides, video**
24	25	26	27	28	29	30
	EMCF26 due at 9am Notes: **page**, **4-per** Homework 9 due in lab/workshop		**EMCF27 due at 9am** Notes: **page**, **4-per** **Homework 10 posted** **Last day to drop with a W**		**EMCF28 due at 9am** Blank Slides: **page**, **4-per** Quiz in lab/workshop **Online Review from 4-6pm page, 4-per**	**Quiz 9 closes** (10.1-10.3) **Test 3 starts** **Check the dates on CourseWare**
31	April 1 **EMCF29 due at 9am**	2	3 Homework 10 due in lab/workshop	4	5	6 **Quiz 10 closes** (10.4-10.5)

Online Review Today from 4-6pm, *but I honestly do not know why am making the **effort***. See the next two pages...

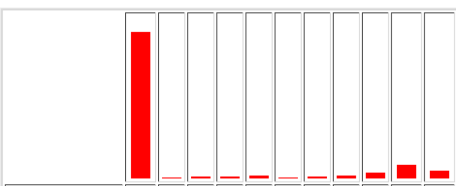
Your Effort on Practice Test 3



number of students	479	7	10	10	4	5	11	11	14	11	16
score range	0 to 10	10 to 20	20 to 30	30 to 40	40 to 50	50 to 60	60 to 70	70 to 80	80 to 90	90 to 100	100 to 110

This is NOT part of the formula for success.

Your Effort on Quiz 9



number of students	452	2	5	6	8	4	6	10	19	42	24
score range	0 to 10	10 to 20	20 to 30	30 to 40	40 to 50	50 to 60	60 to 70	70 to 80	80 to 90	90 to 100	100 to 110

This is NOT part of the formula for success.

Popper 21

Improper Integrals

1. 5/3

Trouble: $x^{-4/3}$ is discont on $[-1,1]$.

Motivating Example: $\displaystyle\int_{-1}^{1} x^{-4/3}\,dx \;\cancel{=}\; -3x^{-1/3}\Big|_{-1}^{1}$

$= -3(1 - -1) = -6$

Note: $f(x) = \dfrac{1}{x^{4/3}}$

$= \left(\dfrac{1}{x^{1/3}}\right)^4 > 0$

This must be wrong!!

Hm... So $\displaystyle\int_{-1}^{1} x^{-4/3}\,dx$ should be positive.

We used the F Thm Calc incorrectly! The integrand must be continuous to use anti-differentiation.

Remark: **The previous integral is referred to as an "Improper" Integral.**

Types of Improper Integrals

$$\int_a^b f(x)dx \qquad \int_{-\infty}^b f(x)dx \qquad \int_a^\infty f(x)dx$$

$$\int_{-\infty}^\infty f(x)dx$$

where a limit of integration is infinite and/or the function f has a discontinuity on the interval.

Proper Notation/Computation

(of improper integrals)

Popper 21

2. –43

$$\int_a^b f(x)dx$$

Assume f is continuous except at $x = c$, where c lies in the interval $[a,b]$.

Case 1: $a < c < b$

Break the integral into 2 pieces.

$$\int_a^{\boxed{c}} f(x)dx + \int_{\boxed{c}}^b f(x)dx = \lim_{\tau\to c^-}\int_a^\tau f(x)dx + \lim_{t\to c^+}\int_t^b f(x)dx$$

Recall: $\int_{-1}^1 x^{-4/3}dx = \lim_{\tau\to 0^-}\int_{-1}^\tau x^{-4/3}dx + \lim_{t\to 0^+}\int_t^1 x^{-4/3}dx$

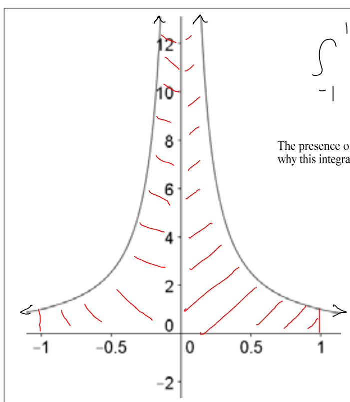
bad at $x = 0$ FThm ok ditto

$$= \lim_{\tau\to 0^-}\left(-3x^{-1/3}\right)\Big|_{-1}^{\tau} + \lim_{t\to 0^+}\left(-3x^{-1/3}\right)\Big|_t^1$$

$$= \lim_{\tau\to 0^-}\left(-3\tau^{-1/3} - 3\right) + \lim_{t\to 0^+}\left(-3 + 3t^{-1/3}\right)$$

$$= \infty + \infty$$

dne



$$\int_{-1}^1 x^{-4/3}dx = \infty$$

The presence of the vertical asymptote is not the only reason why this integral is infinite.

Similar example:

$$\int_{-1}^1 x^{-2/3}dx$$

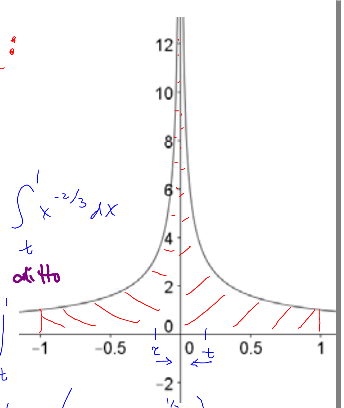
$$= \lim_{\tau\to 0^-}\int_{-1}^\tau x^{-2/3}dx + \lim_{t\to 0^+}\int_t^1 x^{-2/3}dx$$

FThm ok ditto

$$= \lim_{\tau\to 0^-} 3x^{1/3}\Big|_{-1}^{\tau} + \lim_{t\to 0^+} 3x^{1/3}\Big|_t^1$$

$$= \lim_{\tau\to 0^-}\left(3\tau^{1/3} - -3\right) + \lim_{t\to 0^+}\left(3 - 3t^{1/3}\right)$$

$$= 3 + 3 = 6.$$



How do we handle $\int_{-\infty}^{\infty} f(x)\,dx$, $\int_{-\infty}^{b} f(x)\,dx$, $\int_{a}^{\infty} f(x)\,dx$

$\lim_{t\to-\infty} \int_t^b f(x)\,dx$

when f is continuous?

Popper Number 21

3. 0.222

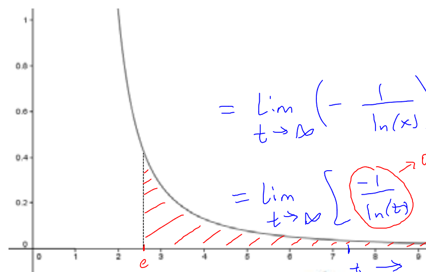
e.g. $\lim_{t\to\infty} \int_a^t f(x)\,dx$

split and use 2 limits.

Example: $\int_e^{\infty} \frac{1}{x(\ln(x))^2}\,dx = \lim_{t\to\infty} \underbrace{\int_e^t \frac{1}{x(\ln(x))^2}\,dx}_{\text{F Thm ok}}$

$= \lim_{t\to\infty} \left(-\frac{1}{\ln(x)}\right)\Big|_e^t$

$= \lim_{t\to\infty} \left[\frac{-1}{\ln(t)} + \frac{1}{\ln(e)}\right] = 1.$



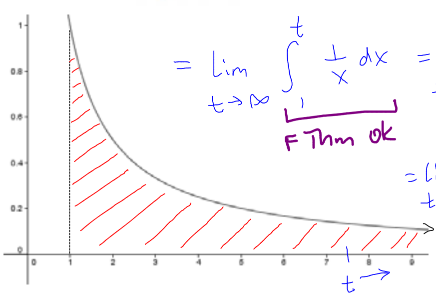
A portion of the graph of $\frac{1}{x(\ln(x))^2}$.

Example: $\int_1^{\infty} \frac{1}{x}\,dx$ DNE

$= \lim_{t\to\infty} \underbrace{\int_1^t \frac{1}{x}\,dx}_{\text{F Thm ok}} = \lim_{t\to\infty} \ln(x)\Big|_1^t$

$= \lim_{t\to\infty} (\ln(t) - 0)$

$= \infty$

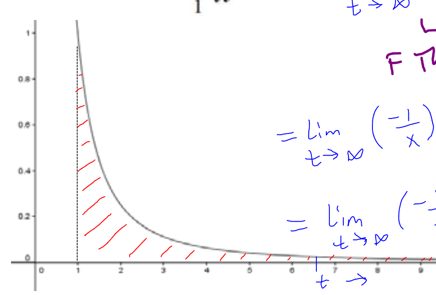


A portion of the plot of $y = 1/x$.

Example: $\int_1^{\infty} \frac{1}{x^2}\,dx = \lim_{t\to\infty} \underbrace{\int_1^t \frac{1}{x^2}\,dx}_{\text{F Thm ok}}$

$= \lim_{t\to\infty} \left(\frac{-1}{x}\right)\Big|_1^t$

$= \lim_{t\to\infty} \left(-\frac{1}{t} + 1\right) = 1$



A portion of the plot of $y = 1/x^2$.

Matt

Improper

An Important Integral...

$$\int_1^{\infty} \frac{1}{x^p}\,dx = \underbrace{\lim_{t\to\infty} \int_1^t \frac{1}{x^p}\,dx}_{\text{F Thm ok}}$$

$$= \begin{cases} \lim_{t\to\infty} \ln(x)\Big|_1^t & p=1 \\ \lim_{t\to\infty} \frac{1}{-p+1} x^{-p+1}\Big|_1^t & p\neq 1 \end{cases}$$

$$= \begin{cases} \infty & p=1 \\ \lim_{t\to\infty} \frac{1}{-p+1}\left(t^{-p+1} - 1\right) & , p\neq 1 \end{cases}$$

$t^{-p+1} \to \infty$ if $p<1$

$t^{-p+1} \to 0$ if $p>1$

$$\int_1^{\infty} \frac{1}{x^p}\,dx = \begin{cases} \infty & p\leq 1 \\ \frac{1}{p-1} & p>1 \end{cases}$$