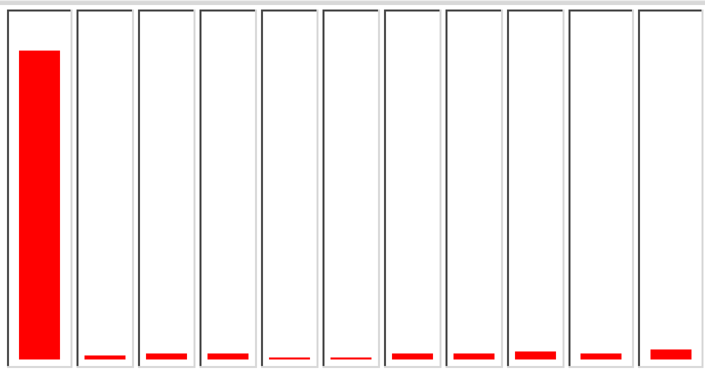
# Information

Take care of **Practice Test 3** ASAP

| 17 | 18 | 19 | 20 | 21 | 22 | 23 |
|---|---|---|---|---|---|---|
| **Practice Test 3 is available** | **EMCF23 due at 9am**<br>Notes: **page**, **4-per**<br>**See Wednesday's Video**<br>Homework 8 due | | **EMCF24 due at 9am**<br>Notes: **page**, **4-per**, **video notes**, **video**<br>**No Office Hours Today**<br>**Homework 9 posted** | | **EMCF25 due at 9am**<br>Notes: **page**, **4-per**, **video notes**, **video**<br>Quiz in lab/workshop | **Quiz 8 closes** (9.6-9.8)<br>**2011 Test 3 Review: slides, video**<br>**2012 Test 3 Review: slides, video** |
| **24** | **25** | **26** | **27** | **28** | **29** | **30** |
| | **EMCF26 due at 9am**<br>Notes: **page**, **4-per**<br>Homework 9 due in lab/workshop | | **EMCF27 due at 9am**<br>Notes: **page**, **4-per**<br>**Homework 10 posted**<br>**Last day to drop with a W** | | **EMCF28 due at 9am**<br>Blank Slides: **page**, **4-per**<br>Quiz in lab/workshop<br>**Online Review from 4-6pm**<br>**page**, **4-per** | **Quiz 9 closes** (10.1-10.3)<br>**Test 3 starts**<br>**Check the dates on CourseWare** |
| **31** | **April 1** | **2** | **3** | **4** | **5** | **6** |
| | **EMCF29 due at 9am** | | Homework 10 due in lab/workshop | | | **Quiz 10 closes** (10.4-10.5) |

**Online Review** Today from 4-6pm, *but I honestly do not know why am making the **effort***. See the next two pages...

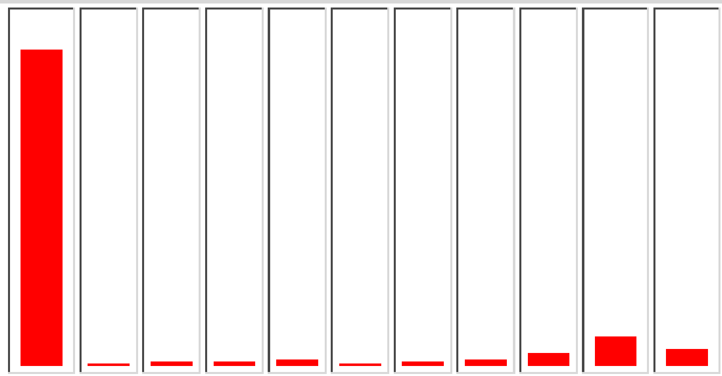
# Your Effort on Practice Test 3



| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| number of students | 479 | 7 | 10 | 10 | 4 | 5 | 11 | 11 | 14 | 11 | 16 |
| score range | 0 to 10 | 10 to 20 | 20 to 30 | 30 to 40 | 40 to 50 | 50 to 60 | 60 to 70 | 70 to 80 | 80 to 90 | 90 to 100 | 100 to 110 |

**This is NOT part of the formula for success.**

# Your Effort on Quiz 9



| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| number of students | 452 | 2 | 5 | 6 | 8 | 4 | 6 | 10 | 19 | 42 | 24 |
| score range | 0 to 10 | 10 to 20 | 20 to 30 | 30 to 40 | 40 to 50 | 50 to 60 | 60 to 70 | 70 to 80 | 80 to 90 | 90 to 100 | 100 to 110 |

**This is NOT part of the formula for success.**

**Popper 21**

# Improper Integrals

1. 5/3

Motivating Example: $\int_{-1}^{1} x^{-4/3}\,dx$

Trouble: $x^{-4/3}$ is discont on $[-1,1]$.

$= -3x^{-1/3}\Big|_{-1}^{1}$

$= -3(1 - -1) = -6$

Note: $f(x) = \frac{1}{x^{4/3}}$

$= \left(\frac{1}{x^{1/3}}\right)^4 > 0$

This must be wrong!!

Hm... So $\int_{-1}^{1} x^{-4/3}\,dx$ should be positive.

We used the F Thm Calc incorrectly! The integrand must be continuous to use anti-differentiation.

**Remark: The previous integral is referred to as an "Improper" Integral.**

# Types of Improper Integrals

$$\int_a^b f(x)dx \qquad \int_{-\infty}^b f(x)dx \qquad \int_a^{\infty} f(x)dx$$

$$\int_{-\infty}^{\infty} f(x)dx$$

where a limit of integration is infinite and/or the function $f$ has a discontinuity on the interval.

# Proper Notation/Computation

(of improper integrals)

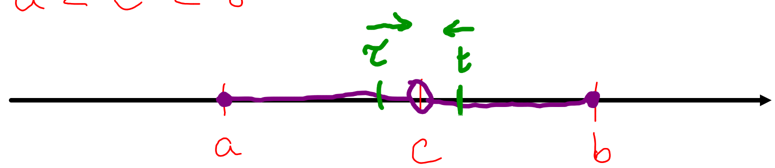
**Popper 21**

2. $-43$

$$\int_a^b f(x)dx$$

Assume $f$ is continuous except at $x = c$, where $c$ lies in the interval $[a,b]$.

Case 1: $a < c < b$

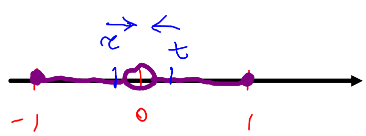



Break the integral into 2 pieces.

$$\int_a^{\boxed{c}} f(x)\,dx + \int_{\boxed{c}}^b f(x)\,dx = \lim_{\tau\to c^-}\int_a^{\tau} f(x)\,dx + \lim_{t\to c^+}\int_t^b f(x)\,dx$$

Recall: $\displaystyle\int_{-1}^{1} x^{-4/3}\,dx = \lim_{\tau\to 0^-}\underbrace{\int_{-1}^{\tau} x^{-4/3}\,dx}_{\text{FThm OK}} + \lim_{t\to 0^+}\underbrace{\int_{t}^{1} x^{-4/3}\,dx}_{\text{ditto}}$

bad at $x=0$

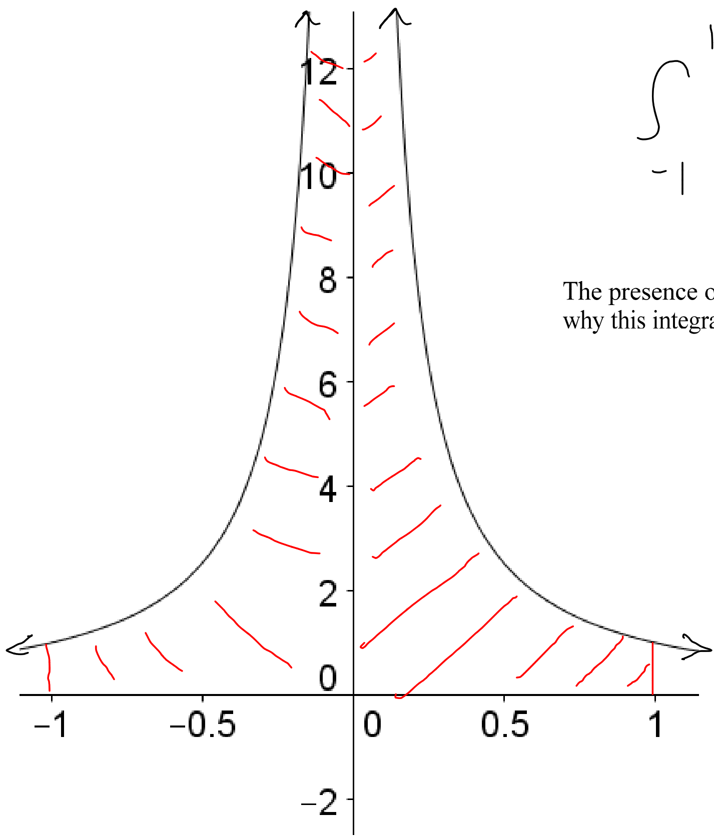



$$= \lim_{\tau\to 0^-}\left(-3x^{-1/3}\right)\Big|_{-1}^{\tau} + \lim_{t\to 0^+}\left(-3x^{-1/3}\right)\Big|_{t}^{1}$$

$$= \underbrace{\lim_{\tau\to 0^-}\left(-3\tau^{-1/3} - 3\right)}\; + \lim_{t\to 0^+}\left(-3 + 3t^{-1/3}\right)$$

$$= \infty \quad + \quad \infty$$

dne




$$\int_{-1}^{1} x^{-4/3}\,dx = \infty$$

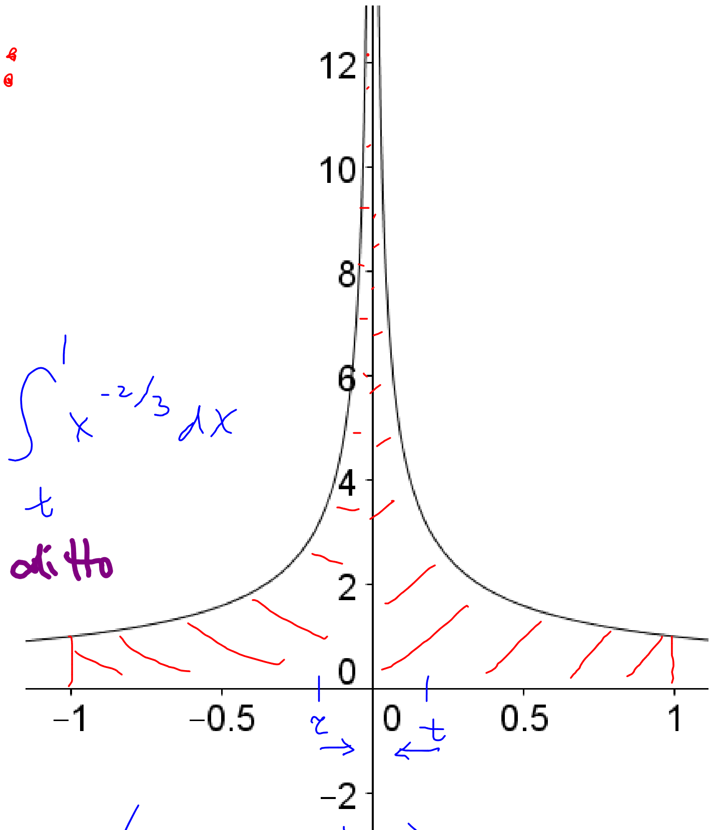
The presence of the vertical asymptote is not the only reason why this integral is infinite.

Similar example:

$$\int_{-1}^{1} x^{-2/3}\,dx$$

$$= \lim_{\tau\to 0^-}\int_{-1}^{\tau} x^{-2/3}\,dx + \lim_{t\to 0^+}\int_{t}^{1} x^{-2/3}\,dx$$

FThm ok     ditto



$$= \lim_{\tau\to 0^-} 3x^{1/3}\Big|_{-1}^{\tau} + \lim_{t\to 0^+} 3x^{1/3}\Big|_{t}^{1}$$

$$= \lim_{\tau\to 0^-}\left(3\underbrace{\tau^{1/3}}_{\to 0} - -3\right) + \lim_{t\to 0^+}\left(3 - 3\underbrace{t^{1/3}}_{\to 0}\right)$$

$$= 3 + 3 = 6.$$

How do we handle

$$\lim_{t \to -\infty} \int_t^b f(x)\,dx$$

$$\int_{-\infty}^{\infty} f(x)dx, \quad \int_{-\infty}^{b} f(x)dx, \quad \int_{a}^{\infty} f(x)dx$$

when $f$ is continuous?

**Popper Number 21**

3. 0.222

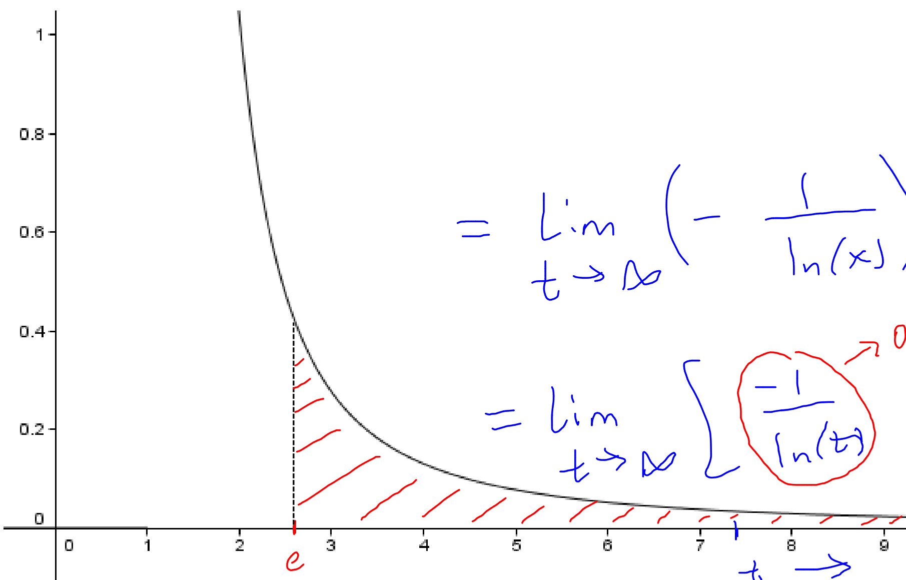
e.g. $\lim_{t \to \infty} \int_a^t f(x)\,dx$

split and use 2 limits.

Example: $\displaystyle\int_e^{\boxed{\infty}} \frac{1}{x(\ln(x))^2}\,dx = \lim_{t\to\infty} \underbrace{\int_e^t \frac{1}{x(\ln(x))^2}\,dx}_{\text{FThm OK}}$

$$= \lim_{t\to\infty} \left(-\frac{1}{\ln(x)}\right)\Bigg|_e^t$$

$$= \lim_{t\to\infty} \left[\frac{-1}{\ln(t)} \xrightarrow{} 0 + \frac{1}{\ln(e)}\right] = 1.$$



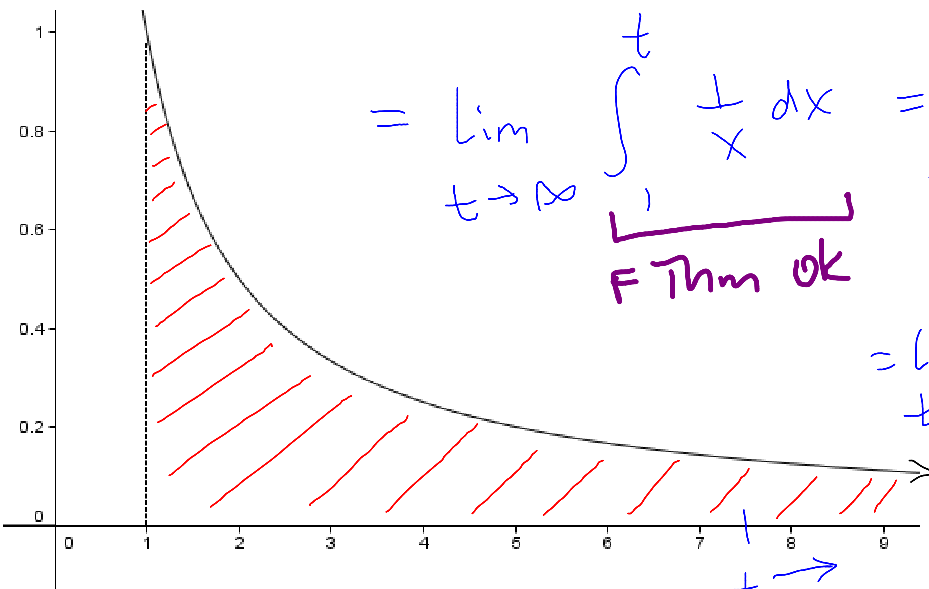

A portion of the graph of $\dfrac{1}{x(\ln(x))^2}$.

Example: $\displaystyle\int_1^{\infty} \frac{1}{x}\,dx$ DNE

$$= \lim_{t\to\infty} \underbrace{\int_1^t \frac{1}{x}\,dx}_{\text{F Thm ok}} = \lim_{t\to\infty} \ln(x)\Big|_1^t$$

$$= \lim_{t\to\infty} \left(\ln(t) - 0\right)$$

$$= \infty$$

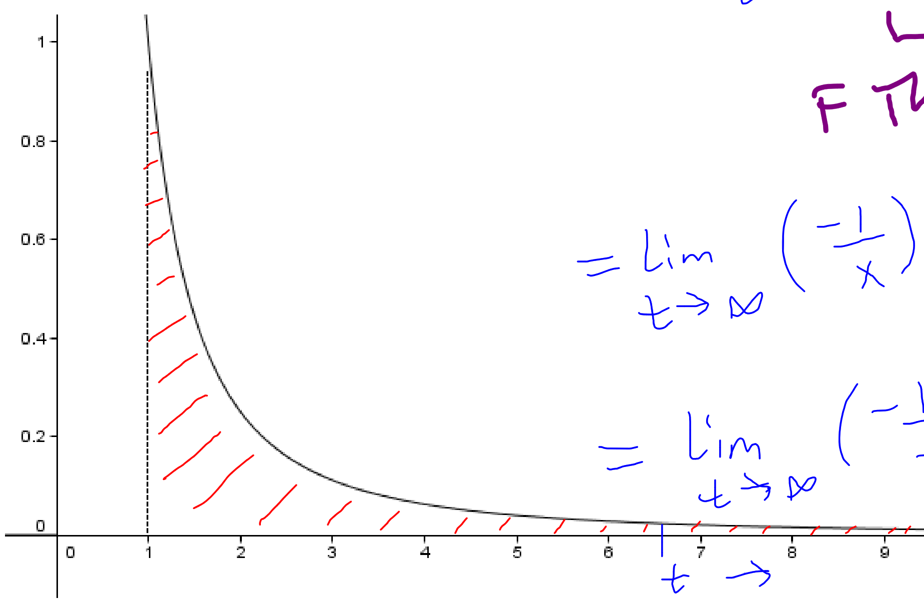



A portion of the plot of $y = 1/x$.

Example: $$\int_1^{\infty} \frac{1}{x^2}\,dx = \lim_{t\to\infty} \underbrace{\int_1^{t} \frac{1}{x^2}\,dx}_{\text{F Thm ok}}$$

$$= \lim_{t\to\infty} \left(\frac{-1}{x}\right)\Bigg|_1^{t}$$

$$= \lim_{t\to\infty} \left(-\frac{1}{t} + 1\right) = 1$$




A portion of the plot of $y = 1/x^2$.

Matt

Improper

# An Important Integral...

$$\int_1^{\infty} \frac{1}{x^p}\,dx = \underbrace{\lim_{t\to\infty} \int_1^t \frac{1}{x^p}\,dx}_{\text{F Thm OK}}$$

$$= \begin{cases} \lim_{t\to\infty} \ln(x)\Big|_1^t & p=1 \\ \lim_{t\to\infty} \frac{1}{-p+1} x^{-p+1}\Big|_1^t & p\neq 1 \end{cases}$$

$$= \begin{cases} \infty & p=1 \\ \lim_{t\to\infty} \frac{1}{-p+1}\left(t^{-p+1} - 1\right), & p \neq 1 \end{cases}$$

$t^{-p+1} \to \infty$ if $p<1$

$t^{-p+1} \to 0$ if $p>1$

$$\int_1^{\infty} \frac{1}{x^p}\,dx = \begin{cases} \infty & p \leq 1 \\ \frac{1}{p-1} & p>1 \end{cases}$$