Comments on Course Grades:

0 ——————————————— Target 800

Remaining Points: ~ 450 + Test 3

- Test 4 - 100
- Final Exam - 200
- Your lowest test score will be replaced with the percentage grade on your final exam (provided it is higher) - 100
- Remaining, Poppers, EMCFs and Recitation Quizzes ~ 50

...Everyone can still make a good grade in this class!!

Popper 22

1. $\displaystyle\int_1^{\infty} \frac{1}{x^3}\,dx$

2. $\displaystyle\int_0^{\infty} \sin^2(x)\,dx$

3. $\displaystyle\int_0^{\pi/2} \cot(x)\sqrt{\sin(x)}\,dx$

2

If the integral diverges, give **999** as your answer.

Sums → **Series** ← Chapter 11

Examples:

Summation notation

$$\sum_{n=1}^{4} \frac{1}{n^2} = \text{"the sum from } i = 1 \text{ to } 4 \text{ of } \frac{1}{n^2}\text{"} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16}$$

$$\sum_{n=1}^{6} \frac{(-1)^n}{5n+1} = -\frac{1}{6} + \frac{1}{11} - \frac{1}{16} + \frac{1}{21} - \frac{1}{26} + \frac{1}{31}$$

$$\sum_{n=0}^{5} \left(\frac{1}{2}\right)^n = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32}$$

$$\sum_{n=0}^{\infty} \left(\frac{-1}{2}\right)^n = 1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \frac{1}{32} + \cdots$$

Infinite Series

Infinite Series

$$\sum_{k=1}^{\infty} a_k$$

could be anything

terms in the series

Popper 22

4. 5/2

Warning: Series

Sequence

Some similarities, but mostly very different.

Infinite Series Notation

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \cdots$$

$$\sum_{k=0}^{\infty} b_k = b_0 + b_1 + b_2 + b_3 + \cdots$$

$$\sum_{j=3}^{\infty} c_j = c_3 + c_4 + c_5 + \cdots$$

what does it mean to say "add these forever"

Infinite Series and Sequences of Partial Sums

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \cdots + a_N + a_{N+1} + \cdots$$

partial sums

S_1

S_2

S_3

$\vdots$

S_N

$\vdots$

The sequence of partial sums is $\{S_N\}_{N=1}^{\infty}$.

We say $\sum_{k=1}^{\infty} a_k$ exists iff $\{S_N\}_{N=1}^{\infty}$ converges. In this case

$$\sum_{k=1}^{\infty} a_k = \lim_{N\to\infty} S_N = \lim_{N\to\infty} \sum_{k=1}^{N} a_k$$

We say the series converges (i.e. adds up to a value) if and only if the sequence of partial sums converges. Otherwise, we say the series diverges (i.e. does not add up to a value).

$$S_N = \sum_{j=1}^{N} a_j$$

$$\sum_{j=1}^{\infty} a_j = \lim_{N\to\infty} S_N$$

Example: $\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} + \cdots$

$$= \lim_{N\to\infty} \sum_{n=1}^{N} \frac{1}{n^2}$$

A few partial sums:

$S_1 = 1$, $S_2 = 1 + \frac{1}{4} = \frac{5}{4}$, $S_3 = \frac{5}{4} + \frac{1}{9} = \frac{49}{36}$

$S_4 = \frac{49}{36} + \frac{1}{16} = \frac{205}{144}$, ...

Excel Exploration

Spreadsheet	**VBA**