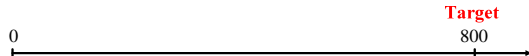


### Comments on Course Grades:



Remaining Points: ~ 450 + Test 3

- Test 4 - 100
- Final Exam - 200
- Your lowest test score will be replaced with the percentage grade on your final exam (provided it is higher) - 100
- Remaining, Poppers, EMCFs and Recitation Quizzes ~ 50

...Everyone can still make a good grade in this class!!

### Popper 22

1.  $\int_1^{\infty} \frac{1}{x^3} dx$     2.  $\int_0^{\infty} \sin^2(x) dx$     3.  $\int_0^{\pi/2} \cot(x) \sqrt{\sin(x)} dx$

2

If the integral diverges, give 999 as your answer.

Sums → Series ← Chapter 11

Examples:

- $\sum_{i=1}^4 \frac{1}{n^2}$  = "the sum from  $i = 1$  to 4 of  $\frac{1}{n^2}$ " =  $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16}$
- $\sum_{n=1}^6 \frac{(-1)^n}{5n+1}$  =  $-\frac{1}{6} + \frac{1}{11} - \frac{1}{16} + \frac{1}{21} - \frac{1}{26} + \frac{1}{31}$
- $\sum_{n=1}^4 \left(\frac{1}{2}\right)^n$  =  $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32}$
- $\sum_{n=0}^{\infty} \left(\frac{-1}{2}\right)^n$  =  $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \frac{1}{32} + \dots$

∞  
↑  
Infinite Series

### Infinite Series

Popper 22

4. 5/2

could be anything →  $\sum_{k=1}^{\infty} a_k$  ← terms in the series

Warning:    Series  
                         Sequence

Some similarities, but mostly very different.

### Infinite Series Notation

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \dots$$

$$\sum_{k=0}^{\infty} b_k = b_0 + b_1 + b_2 + b_3 + \dots$$

$$\sum_{j=3}^{\infty} c_j = c_3 + c_4 + c_5 + \dots$$

What does it mean to say "add these forever"

### Infinite Series and Sequences of Partial Sums

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \dots + a_N + a_{N+1} + \dots$$

Partial sums:  $S_1, S_2, S_3, \dots, S_N, \dots$

The sequence of partial sums is  $\{S_N\}_{N=1}^{\infty}$ .

We say  $\sum_{k=1}^{\infty} a_k$  exists iff  $\{S_N\}_{N=1}^{\infty}$  converges. In this case

$$\sum_{k=1}^{\infty} a_k = \lim_{N \rightarrow \infty} S_N = \lim_{N \rightarrow \infty} \sum_{k=1}^N a_k$$

We say the series converges (i.e. adds up to a value) if and only if the sequence of partial sums converges. Otherwise, we say the series diverges (i.e. does not add up to a value).

$$S_N = \sum_{j=1}^N a_j$$

$$\sum_{j=1}^{\infty} a_j = \lim_{N \rightarrow \infty} S_N$$

Example:  $\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} + \dots$   
 $= \lim_{N \rightarrow \infty} \sum_{n=1}^N \frac{1}{n^2}$

A few partial sums:

$S_1 = 1, S_2 = 1 + \frac{1}{4} = \frac{5}{4}, S_3 = \frac{5}{4} + \frac{1}{9} = \frac{49}{36}$   
 $S_4 = \frac{49}{36} + \frac{1}{16} = \frac{205}{144}, \dots$

Excel Exploration

Spreadsheet

VBA

