# Comments on Course Grades:


**Remaining Points: ~ 450 + Test 3**

- Test 4 - 100
- Final Exam - 200
- Your lowest test score will be replaced with the percentage grade on your final exam (provided it is higher) - 100
- Remaining, Poppers, EMCFs and Recitation Quizzes ~ 50

**...Everyone can still make a good grade in this class!!**

**Popper 22**

1. $\displaystyle\int_1^{\infty} \frac{1}{x^3}\,dx$

2. $\displaystyle\int_0^{\infty} \sin^2(x)\,dx$

3. $\displaystyle\int_0^{\pi/2} \cot(x)\sqrt{\sin(x)}\,dx$

2

If the integral diverges, give **999** as your answer.

Sums → # Series ← Chapter 11

**Examples:** $\sum_{n=1}^{4} \frac{1}{n^2}$ = "the sum from $i = 1$ to 4 of $\frac{1}{n^2}$" $= 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16}$

Summation notation

$$\sum_{n=1}^{6} \frac{(-1)^n}{5n+1} = -\frac{1}{6} + \frac{1}{11} - \frac{1}{16} + \frac{1}{21} - \frac{1}{26} + \frac{1}{31}$$

$$\sum_{n=0}^{5} \left(\frac{1}{2}\right)^n = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32}$$

$$\sum_{n=0}^{\infty} \left(\frac{-1}{2}\right)^n = 1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \frac{1}{32} + \cdots$$

Infinite Series

# Infinite Series

**Popper 22**

4. 5/2

$$\sum_{k=1}^{\infty} a_k$$

could be anything

terms in the series

Warning: Series

Sequence

Some similarities, but mostly very different.

# Infinite Series Notation

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \cdots$$

$$\sum_{k=0}^{\infty} b_k = b_0 + b_1 + b_2 + b_3 + \cdots$$

$$\sum_{j=3}^{\infty} c_j = c_3 + c_4 + c_5 + \cdots$$

what does it mean to say "add these forever"

# Infinite Series and Sequences of Partial Sums

$$\sum_{n=1}^{\infty} a_n = \underbrace{a_1}_{S_1} + a_2 + a_3 + \cdots + a_N + a_{N+1} + \cdots$$

partial sums

$S_1$

$S_2$

$S_3$

$\vdots$

$S_N$

$\vdots$

The sequence of partial sums is $\{S_N\}_{N=1}^{\infty}$.

We say $\sum_{k=1}^{\infty} a_k$ exists iff $\{S_N\}_{N=1}^{\infty}$ converges. In this case

$$\sum_{k=1}^{\infty} a_k = \lim_{N\to\infty} S_N = \lim_{N\to\infty} \sum_{k=1}^{N} a_k$$

We say the series converges (i.e. adds up to a value) if and only if the sequence of partial sums converges. Otherwise, we say the series diverges (i.e. does not add up to a value).

$$S_N = \sum_{j=1}^{N} a_j$$

$$\sum_{j=1}^{\infty} a_j = \lim_{N\to\infty} S_N$$

**Example:** $\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} + \cdots$

$$= \lim_{N \to \infty} \sum_{n=1}^{N} \frac{1}{n^2}$$

**A few partial sums:**

$S_1 = 1$, $S_2 = 1 + \frac{1}{4} = \frac{5}{4}$, $S_3 = \frac{5}{4} + \frac{1}{9} = \frac{49}{36}$

$S_4 = \frac{49}{36} + \frac{1}{16} = \frac{205}{144}$, ...

**Excel Exploration**

| **Spreadsheet** | **VBA** |
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