Series ← chapter 11

terms

Examples: $\boxed{\sum_{n=1}^{4}}\frac{1}{n^2} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2}$

Summation Notation

$$\boxed{\sum_{n=1}^{6}}\frac{(-1)^n}{5n+1} = \frac{-1}{6} + \frac{1}{11} - \frac{1}{16} + \frac{1}{21} - \frac{1}{26} + \frac{1}{31}$$

$$\boxed{\sum_{n=0}^{5}}\left(\frac{1}{2}\right)^n = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32}$$

$$\boxed{\sum_{n=0}^{\infty}}\left(\frac{-1}{2}\right)^n = 1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \frac{1}{32} + \cdots$$

example of an infinite series

Infinite Series

terms

$$\sum_{k=1}^{\infty} a_k \quad \text{or} \quad \sum_{k=0}^{\infty} a_k \quad \text{or} \quad \sum_{k=3}^{\infty} a_k$$

Warning: Series

Sequence

Some similarities, but mostly very different.

Infinite Series Notation

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \cdots$$

$$\sum_{k=0}^{\infty} b_k = b_0 + b_1 + b_2 + \cdots$$

$$\sum_{j=3}^{\infty} c_j = c_3 + c_4 + c_5 + \cdots$$

what does "..." mean?

Infinite Series and Sequences of Partial Sums

$$\sum_{n=1}^{\infty} a_n = \underbrace{\underbrace{\underbrace{a_1}_{S_1} + a_2}_{S_2} + a_3}_{S_3} + \cdots + a_N + a_{N+1} + \cdots$$

partial sums

S_1

S_2

S_3

$\vdots$

S_N

$\vdots$

$\{S_N\}_{N=1}^{\infty}$ is the sequence of partial sums

$$\sum_{n=1}^{\infty} a_n = \lim_{N \to \infty} S_N$$

We say the series converges (i.e. adds up to a value) if and only if the sequence of partial sums converges. Otherwise, we say the series diverges (i.e. does not add up to a value).

$$S_N = \sum_{j=1}^{N} a_j$$

$$\sum_{j=1}^{\infty} a_j = \lim_{N \to \infty} S_N$$

Example: $\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \cdots$

A few partial sums: $S_1 = 1$, $S_2 = \frac{5}{4}$, $S_3 = \frac{5}{4} + \frac{1}{9}$

$= \frac{49}{36}$

you can do S_4, S_5, etc...

Excel Exploration

Spreadsheet

N	S_N
99980	1.644924065
99981	1.644924065
99982	1.644924065
99983	1.644924065
99984	1.644924065
99985	1.644924065
99986	1.644924065
99987	1.644924066
99988	1.644924066
99989	1.644924066
99990	1.644924066
99991	1.644924066
99992	1.644924066
99993	1.644924066
99994	1.644924066
99995	1.644924066
99996	1.644924066
99997	1.644924067
99998	1.644924067
99999	1.644924067
100000	1.644924067

It appears as though the series is converging, because the sequence of partial sums appears to be converging.

VBA

```
function  a(k)
a = 1 / k^2
end function

function s(i, n)
s = 0
for  k = i  to  n
    s = s + a(k)
next k
end function
```