## No Office Hours Today

| 7 | 8 | 9 | 10 | 11 | 12 | 13 |
|---|---|---|---|---|---|---|
| | EMCF32 due at 9am<br>Notes: page, 4-per, video notes, video<br>Homework 11 due in lab/workshop | Extra Series Video | EMCF33 due at 9am<br>Blank Slides: page, 4-per, video notes, video<br>Homework 12 posted | Video explanation of why a conditionally convergent series can sum to any value, and an Excel demonstration. | EMCF34 due at 9am<br>Quiz in lab/workshop<br>Practice Test 4 Opens | Quiz 11 closes (10.6-10.7) |
| 14 | 15<br>EMCF35 due at 9am<br>Homework 12 due in lab/workshop | 16 | 17<br>EMCF36 due at 9am<br>Homework 13 posted | 18 | 19<br>EMCF37 due at 9am<br>Quiz in lab/workshop | 20<br>Quiz 12 closes (11.1-11.4) |
| 21 | 22<br>EMCF38 due at 9am<br>Homework 13 due in lab/workshop | 23 | 24<br>EMCF39 due at 9am<br>Homework 14 posted | 25 | 26<br>EMCF40 due at 9am<br>Quiz in lab/workshop | 27<br>Quiz 13 closes (11.5-11.6)<br>Test 4 starts |

See this for help with Hwk in 11.4.

## Useful Growth Information

Gut

If $\alpha > 0$ and $a > 1$, then

$$\ln(n) << n^{\alpha} << a^n << n! << n^n$$

Comment: If $0 \le a_n << b_n$ and $\sum \frac{1}{b_n}$ converges. Then $\sum \frac{a_n}{b_n}$ converges.

| | Gut | Work |
|---|---|---|
| $\sum_{n=1}^{\infty} \frac{n^{16}}{2^n}$ | $n^{16} << 2^n$ and $\sum \frac{1}{2^n}$ converges ∴ Converges. | To verify: ratio, root |
| $\sum_{n=1}^{\infty} \frac{\ln(n)}{n^2}$ | $\ln(n) << n^2$ and $\sum \frac{1}{n^2}$ converges ∴ Converges | To verify: Idea $0 \le \ln(n) \le \sqrt{n}$ for large $n$. ∴ for large $n$ $0 \le \frac{\ln(n)}{n^2} \le \frac{\sqrt{n}}{n^2} = \frac{1}{n^{3/2}}$ and $\sum \frac{1}{n^{3/2}}$ converges ∴ By the CT, our series converges. |
| $\sum_{n=1}^{\infty} \frac{\ln(n)}{n}$ | Diverges<br>$\frac{\ln(n)}{n} \ge \frac{1}{n}$ for $n \ge 3$ and $\sum \frac{1}{n}$ diverges. ∴ by C.T., our series diverges. | |
| $\sum_{n=1}^{\infty} \frac{3^n}{n!}$ | Gut<br>Converges b/c $3^n << n!$ and $\sum \frac{1}{n!}$ converges. | verify<br>ratio |

## Popper 26

1. Give the value of $\sum_{n=3}^{\infty} \frac{1}{2^n}$.
2. Give the value of $\sum_{n=3}^{\infty} \frac{\cos(n\pi)}{3^n}$.
3. $\sum_{n=1}^{\infty} \frac{2n+1}{n^3+2n}$
   (0) Converges
   (1) Diverges
4. $\sum_{n=1}^{\infty} \frac{3n^4}{2^n}$
   (0) Converges
   (1) Diverges
5. $\sum_{n=1}^{\infty} \frac{n\ln(n)}{n^2+10n+1}$
   (0) Converges
   (1) Diverges

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New

The series $\sum_{k=1}^{\infty} a_k$ **converges absolutely** if and only if $\sum_{k=1}^{\infty} |a_k|$ converges.

Great Convergence

rearrangement invariant

$$\boxed{\text{If } \sum_{k=1}^{\infty} |a_k| \text{ converges, then } \sum_{k=1}^{\infty} a_k \text{ converges.}}$$

(the converse is not true)

The series $\sum_{n=1}^{\infty} a_n$ **converges conditionally** if and only if the series converges, but the series does not converge absolutely.

Crummy Convergence.

arrangement dependent.

New

**Incredible Fact:** If a series is conditionally convergent, and you pick your favorite real number $L$, then you can rearrange the terms in the series so that the sum of the series is $L$. On the other hand, if a series is absolutely convergent, then the sum is the same regardless of the order in which the terms are added.

Example: $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$

I will post a video this afternoon showing how the terms can be rearranged "add up to" any predetermined value.

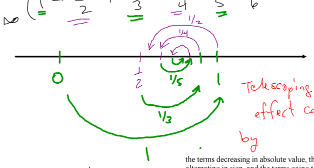
ABS CONV? check $\sum_{n=1}^{\infty} \left| \frac{(-1)^{n+1}}{n} \right|$

$= \sum_{n=1}^{\infty} \frac{1}{n}$ which diverges.

∴ $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ does not converge absolutely.

COND CONV? $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = \lim_{N\to\infty} \sum_{n=1}^{N} \frac{(-1)^{n+1}}{n}$

$= \lim_{N\to\infty} \left( 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \cdots + \frac{(-1)^{N+1}}{N} \right)$



Telescoping effect caused by the terms decreasing in absolute value, the terms alternating in sign, and the terms going to zero.

Converges!

∴ $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ converges conditionally.

Crummy Convergence

The structure in the preceding examples occurs so often that the corresponding series have a special name:

An alternating series is one that can be written in the form

$$\sum_{k=1}^{\infty} (-1)^k a_k \quad \text{or} \quad \sum_{k=1}^{\infty} (-1)^{k+1} a_k \quad \text{with } a_k \ge 0.$$

can be anything

Alternating Series Test: Suppose $a_k \ge 0$ for all $k$, $\{a_k\}$ is a decreasing sequence, and $a_k \to 0$ as $k \to \infty$. Then both

$$\sum_{k=1}^{\infty} (-1)^k a_k \quad \text{and} \quad \sum_{k=1}^{\infty} (-1)^{k+1} a_k \quad \text{converge.}$$

does not matter

**Question:** How close is $S_N$ to the full sum?

$a_N$

(provided we start at $k=1$)

Example: $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{\sqrt{2k+1}}$

Determine whether the series converges absolutely or conditionally.

ABS CONV? Check $\sum_{k=1}^{\infty} \left| \frac{(-1)^{k+1}}{\sqrt{2k+1}} \right|$

$= \sum_{k=1}^{\infty} \frac{1}{\sqrt{2k+1}}$

diverges by LCT with $\sum \frac{1}{k^{1/2}}$.

∴ This series does not converge absolutely.

COND CONV? $\sum_{k=1}^{\infty} (-1)^{k+1} \underbrace{\frac{1}{\sqrt{2k+1}}}_{a_k}$

This is an alternating series.

Note: $a_k = \frac{1}{\sqrt{2k+1}}$ are nonnegative

and $a_k$ decrease

and $a_k \to 0$ as $k \to \infty$.

∴ by the A.S.T. our series converges.

⇒ The series converges conditionally.

Crummy Convergence

Example: $\sum_{k=1}^{\infty} \frac{(-1)^k (k+1)}{k^3}$

Determine whether the series converges absolutely or conditionally.

ABS CONV? Check $\sum \left| \frac{(-1)^k (k+1)}{k^3} \right|$

$= \sum \frac{k+1}{k^3}$

converges by LCT with $\sum \frac{1}{k^2}$.

$\therefore$ our series converges absolutely.

Great Convergence!

## Exercises from 11.4:

See the video!!

5. $\sum (-1)^k \frac{\ln k}{k}$. ABS ✗ COND ✓

6. $\sum (-1)^k \frac{k}{\ln k}$. Diverges terms $\not\to 0$.

7. $\sum \left(\frac{1}{k} - \frac{1}{k!}\right)$. Conv.

8. $\sum \frac{k^3}{2^k}$. Conv (ratio)

9. $\sum (-1)^k \frac{1}{2k+1}$. ABS ✗ COND ✓

10. $\sum (-1)^k \frac{(k!)^2}{(2k)!}$.

11. $\sum \frac{k!}{(-2)^k}$. Diverges terms $\not\to 0$

12. $\sum \sin\left(\frac{k\pi}{4}\right)$. diverges terms $\not\to 0$.

13. $\sum (-1)^k \left(\sqrt{k+1} - \sqrt{k}\right)$.

14. $\sum (-1)^k \frac{k}{k^2+1}$.

15. $\sum \sin\left(\frac{\pi}{4k^2}\right)$.

16. $\sum \frac{(-1)^k}{\sqrt{k(k+1)}}$.

17. $\sum (-1)^k \frac{k}{2^k}$.

18. $\sum \left(\frac{1}{\sqrt{k}} - \frac{1}{\sqrt{k+1}}\right)$

19. $\sum \frac{(-1)^k}{k - 2\sqrt{k}}$.

20. $\sum (-1)^k \frac{k+2}{k^2+k}$.

## Exercises from 11.4:

See the video!!

21. $\sum (-1)^k \frac{4^{k-2}}{e^k}$.

22. $\sum (-1)^k \frac{k^2}{2^k}$.

23. $\sum (-1)^k k \sin(1/k)$.

24. $\sum (-1)^{k+1} \frac{k^k}{k!}$.

25. $\sum (-1)^k k e^{-k}$.

26. $\sum \frac{\cos \pi k}{k}$.

27. $\sum (-1)^k \frac{\cos \pi k}{k}$.

28. $\sum \frac{\sin(\pi k/2)}{k\sqrt{k}}$.

29. $\sum \frac{\sin(\pi k/4)}{k^2}$.

## Exercises from Chapter 11 Highlights:

49. $\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)}$

50. $\sum_{k=0}^{\infty} \frac{k+1}{3^k}$

51. $\sum_{k=0}^{\infty} \frac{(-1)^k (100)^k}{k!}$

52. $\sum_{k=0}^{\infty} \frac{k + \cos k}{k^3+1}$

53. $\sum_{k=0}^{\infty} \frac{(-1)^k}{\sqrt{(k+1)(k+2)}}$

54. $\sum_{k=1}^{\infty} k \left(\frac{3}{4}\right)^k$

55. $\sum_{k=0}^{\infty} \frac{k^e}{e^k}$

56. $\sum_{k=1}^{\infty} (-1)^{k-1} \frac{\ln k}{\sqrt{k}}$

57. $\sum_{k=0}^{\infty} \frac{(2k)!}{2^k k!}$

58. $\sum_{k=0}^{\infty} \frac{(-1)^k}{\sqrt{k^3+1}}$

59. $\sum_{k=0}^{\infty} \frac{(\tan^{-1} k)^2}{1+k^2}$

60. $\sum_{k=0}^{\infty} \frac{2^k + k^4}{3^k}$

See the video!!