No Office Hours Today

7	8 **EMCF32 due at 9am** Notes: **page**, **4-per**, **video notes**, **video** Homework 11 due in lab/workshop	9 **Extra Series Video**	10 **EMCF33 due at 9am** Blank Slides: **page**, **4-per**, **video notes**, **video** **Homework 12 posted**	11 **Video explanation of why a conditionally convergent series can sum to any value, and an Excel demonstration.**	12 **EMCF34 due at 9am** Quiz in lab/workshop **Practice Test 4 Opens**	13 **Quiz 11 closes** (10.6-10.7)
14	15 EMCF35 due at 9am Homework 12 due in lab/workshop	16	17 EMCF36 due at 9am Homework 13 posted	18	19 EMCF37 due at 9am Quiz in lab/workshop	20 **Quiz 12 closes** (11.1-11.4)
21	22 EMCF38 due at 9am Homework 13 due in lab/workshop	**23**	24 EMCF39 due at 9am Homework 14 posted	25	26 EMCF40 due at 9am Quiz in lab/workshop	27 **Quiz 13 closes** (11.5-11.6) **Test 4 starts**

See this for help with the work in 11.4.

Useful Growth Information

Gut: Comment: If $0 \le a_n << b_n$ and $\sum \frac{1}{b_n}$ converges. Then $\sum \frac{a_n}{b_n}$ converges.

> If $\alpha > 0$ and $a > 1$, then
>
> $\ln(n) << n^\alpha << a^n << n! << n^n$

	Gut	Work
$\sum_{n=1}^{\infty} \frac{n^{16}}{2^n}$	$n^{16} << 2^n$ and $\sum \frac{1}{2^n}$ converges $\therefore$ converges.	To verify: ratio, root
$\sum_{n=1}^{\infty} \frac{\ln(n)}{n^2}$	$\ln(n) << n^2$ and $\sum \frac{1}{n^2}$ converges $\therefore$ Converges	To verify: Idea $0 \le \ln(n) \le \sqrt{n}$ for large n. $\therefore$ for large n $0 \le \frac{\ln(n)}{n^2} \le \frac{\sqrt{n}}{n^2} = \frac{1}{n^{3/2}}$ and $\sum \frac{1}{n^{3/2}}$ converges $\therefore$ By the CT, our series converges.
$\sum_{n=1}^{\infty} \frac{\ln(n)}{n}$	Diverges. $\frac{\ln(n)}{n} \ge \frac{1}{n}$ for $n \ge 3$ and $\sum \frac{1}{n}$ diverges. $\therefore$ by C.T., our series diverges.	
$\sum_{n=1}^{\infty} \frac{3^n}{n!}$		

Gut: Converges b/c $3^n << n!$ and $\sum \frac{1}{n!}$ converges.

Verify: ratio

Popper 26

1. Give the value of $\sum_{n=3}^{\infty} \frac{1}{2^n}$.

2. Give the value of $\sum_{n=3}^{\infty} \frac{\cos(n\pi)}{3^n}$.

3. $\sum_{n=1}^{\infty} \frac{2n+1}{n^3+2n}$

   (0) Converges

   (1) Diverges

4. $\sum_{n=1}^{\infty} \frac{3n^4}{2^n}$

   (0) Converges

   (1) Diverges

5. $\sum_{n=1}^{\infty} \frac{n\ln(n)}{n^2+10n+1}$

   (0) Converges

   (1) Diverges

New

rearrangement invariant

The series $\sum_{k=1}^{\infty} a_k$ converges absolutely if and only if

$$\sum_{k=1}^{\infty} |a_k| \text{ converges.}$$

Great Convergence

$$\boxed{\text{If } \sum_{k=1}^{\infty} |a_k| \text{ converges, then } \sum_{k=1}^{\infty} a_k \text{ converges.}}$$

(the converse is not true)

The series $\sum_{n=1}^{\infty} a_n$ converges conditionally if and only if the series converges, but the series does not converge absolutely.

Crummy Convergence.

arrangement dependent.

New

Incredible Fact: If a series is conditionally convergent, and you pick your favorite real number L, then you can rearrange the terms in the series so that the sum of the series is L. On the other hand, if a series is absolutely convergent, then the sum is the same regardless of the order in which the terms are added.

Example: $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$

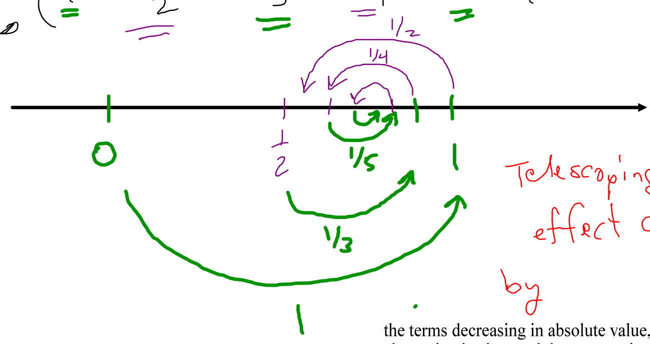
I will post a video this afternoon showing how the terms can be rearranged "add up to" any predetermined value.

ABS CONV? Check $\displaystyle\sum_{n=1}^{\infty} \left| \frac{(-1)^{n+1}}{n} \right| = \sum_{n=1}^{\infty} \frac{1}{n}$ which diverges.

$\therefore \displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ does not converge absolutely.

COND CONV? $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = \lim_{N\to\infty} \sum_{n=1}^{N} \frac{(-1)^{n+1}}{n}$

$$= \lim_{N\to\infty} \left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \cdots + \frac{(-1)^{N+1}}{N} \right)$$



Telescoping effect caused by the terms decreasing in absolute value, the terms alternating in sign, and the terms going to zero.

Converges!

$\therefore \displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ converges conditionally.

Crummy Convergence

The structure in the preceding example~~s~~ occurs so often that the corresponding series have a special name:

An alternating series is one that can be written in the form

$$\sum_{k=1}^{\infty}(-1)^k a_k \quad \text{or} \quad \sum_{k=1}^{\infty}(-1)^{k+1} a_k \text{ with } a_k \geq 0.$$

can be anything

Alternating Series Test: Suppose $a_k \geq 0$ for all k, $\{a_k\}$ is a decreasing sequence, and $a_k \to 0$ as $k \to \infty$. Then both

$$\sum_{k=1}^{\infty}(-1)^k a_k \text{ and } \sum_{k=1}^{\infty}(-1)^{k+1} a_k \text{ converge.}$$

does not matter

Question: How close is S_N to the full sum?

a_N

(provided we start at $k=1$)

Example: $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{\sqrt{2k+1}}$

Determine whether the series converges absolutely or conditionally.

ABS CONV? Check $\sum_{k=1}^{\infty} \left| \frac{(-1)^{k+1}}{\sqrt{2k+1}} \right|$

$= \boxed{\sum_{k=1}^{\infty} \frac{1}{\sqrt{2k+1}}}$

diverges by LCT with $\sum \frac{1}{k^{1/2}}$.

$\therefore$ This series does not converge absolutely.

COND CONV? $\sum_{k=1}^{\infty} (-1)^{k+1} \underbrace{\frac{1}{\sqrt{2k+1}}}_{a_k}$

This is an alternating series.

Note: $a_k = \frac{1}{\sqrt{2k+1}}$ are nonnegative

and a_k decrease

and $a_k \to 0$ as $k \to \infty$.

$\therefore$ by the A.S.T. our series converges.

$\Rightarrow$ the series converges conditionally.

Crummy Convergence

Example: $\sum_{k=1}^{\infty} \frac{(-1)^k (k+1)}{k^3}$

Determine whether the series converges absolutely or conditionally.

ABS CONV? Check $\sum \left| \frac{(-1)^k (k+1)}{k^3} \right|$

$= \sum \frac{k+1}{k^3}$

converges by LCT with $\sum \frac{1}{k^2}$.

$\therefore$ our series converges absolutely.

Great Convergence

Exercises from 11.4:

See the video!!

5. $\sum (-1)^k \frac{\ln k}{k}$. ABS × COND ✓

6. $\sum (-1)^k \frac{k}{\ln k}$. Diverges terms $\not\to 0$.

7. $\sum \left(\frac{1}{k} - \frac{1}{k!}\right)$. Conv.

8. $\sum \frac{k^3}{2^k}$. Conv ratio

9. $\sum (-1)^k \frac{1}{2k+1}$. ABS × COND ✓

10. $\sum (-1)^k \frac{(k!)^2}{(2k)!}$.

11. $\sum \frac{k!}{(-2)^k}$. Diverges terms $\not\to 0$

12. $\sum \sin\left(\frac{k\pi}{4}\right)$. diverges terms $\not\to 0$.

13. $\sum (-1)^k \left(\sqrt{k+1} - \sqrt{k}\right)$.

14. $\sum (-1)^k \frac{k}{k^2+1}$.

15. $\sum \sin\left(\frac{\pi}{4k^2}\right)$.

16. $\sum \frac{(-1)^k}{\sqrt{k(k+1)}}$.

17. $\sum (-1)^k \frac{k}{2^k}$.

18. $\sum \left(\frac{1}{\sqrt{k}} - \frac{1}{\sqrt{k+1}}\right)$

19. $\sum \frac{(-1)^k}{k - 2\sqrt{k}}$.

20. $\sum (-1)^k \frac{k+2}{k^2+k}$.

Exercises from 11.4:

See the video!!

21. $\sum (-1)^k \dfrac{4^{k-2}}{e^k}$.

22. $\sum (-1)^k \dfrac{k^2}{2^k}$.

23. $\sum (-1)^k k \sin(1/k)$.

24. $\sum (-1)^{k+1} \dfrac{k^k}{k!}$.

25. $\sum (-1)^k k e^{-k}$.

26. $\sum \dfrac{\cos \pi k}{k}$.

27. $\sum (-1)^k \dfrac{\cos \pi k}{k}$.

28. $\sum \dfrac{\sin(\pi k/2)}{k\sqrt{k}}$.

29. $\sum \dfrac{\sin(\pi k/4)}{k^2}$.

Exercises from Chapter 11 Highlights:

49. $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)}$

50. $\displaystyle\sum_{k=0}^{\infty} \frac{k+1}{3^k}$

51. $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k (100)^k}{k!}$

52. $\displaystyle\sum_{k=0}^{\infty} \frac{k + \cos k}{k^3+1}$

53. $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k}{\sqrt{(k+1)(k+2)}}$

54. $\displaystyle\sum_{k=1}^{\infty} k \left(\frac{3}{4}\right)^k$

55. $\displaystyle\sum_{k=0}^{\infty} \frac{k^e}{e^k}$

56. $\displaystyle\sum_{k=1}^{\infty} (-1)^{k-1} \frac{\ln k}{\sqrt{k}}$

57. $\displaystyle\sum_{k=0}^{\infty} \frac{(2k)!}{2^k k!}$

58. $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k}{\sqrt{k^3+1}}$

59. $\displaystyle\sum_{k=0}^{\infty} \frac{(\tan^{-1} k)^2}{1+k^2}$

60. $\displaystyle\sum_{k=0}^{\infty} \frac{2^k + k^4}{3^k}$

See the video!!