Test 4 Study Materials: A video review and additional practice problems have been posted.

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Day

Taylor Polynomial Approximations

Most functions have graphs that locally look like polynomials.

Question: Can we approximate a function if we know the function completely at a single point?

i.e., you know the function value and all derivative values at this point.

- $f(x) = e^x$ at $x = 0$
- $g(x) = \sin(x)$ at $x = 0$
- $h(x) = \cos(x)$ at $x = 0$
- $G(x) = \frac{1}{x}$ at $x = 1$

...
**Goal:** Given a function \( f \), and a value \( x = a \) where we know \( f \) and its derivatives, give a polynomial that approximates \( f \).

**The Main Idea of Taylor Polynomials:** Given a function \( f \), and a value \( x = a \) where we know \( f \) and its derivatives, give a polynomial that approximates \( f \).

**How:** Suppose \( p_k(x) \) is a polynomial of degree \( n \).

**Write:**

\[
p(x) = p_k(x) = c_0 + c_1(x-a) + c_2(x-a)^2 + \cdots + c_k(x-a)^k
\]

\[
p'(x) = p'(x) = c_1 + 2c_2(x-a) + 3c_3(x-a)^2 + \cdots
\]

\[
p''(x) = p''(x) = 2c_2 + 6c_3(x-a) + \cdots
\]

\[
p^{(k)}(x) = p^{(k)}(x) = 2c_2 + 6c_3(x-a) + \cdots
\]

**In general ...**

\[
c_k = \frac{1}{k!} p^{(k)}(a)
\]

**The general process...**

Given \( f(x) \), a value \( x = a \), and a positive integer \( n \), find an \( n \)th degree polynomial \( p_n(x) \) so that

\[
p_n(a) = f(a), \quad p_n'(a) = f'(a), \ldots, p_n^{(n)}(a) = f^{(n)}(a)
\]

The Taylor polynomial approximation of \( f \) of degree \( n \) centered at \( x = a \) is given by

\[
p_n(x) = f(a) + \sum_{k=1}^{n} \frac{f^{(k)}(a)}{k!} (x-a)^k
\]

\[
= f(a) + \frac{f'(a)}{1!} (x-a) + \frac{f''(a)}{2!} (x-a)^2 + \cdots
\]

\[
+ \frac{f^{(n)}(a)}{n!} (x-a)^n
\]
Examples:

Give the 5th degree Taylor polynomial centered at 0 for each of \( e^x \), \( \cos(x) \), \( \sin(x) \), \( 1 + \frac{1}{1-x} \) and \( \ln(x+1) \).

1. \( f(x) = e^x \)  \( \Rightarrow \) \[ f^n(x) = e^x \] \( \Rightarrow \) \[ f^n(0) = 1 \] ; \( P_5(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} \)

2. \( f(x) = \cos(x) \)  \( \Rightarrow \) \[ f^n(x) = \cos(x) \] \( \Rightarrow \) \[ f^n(0) = 1 \] ; \( P_5(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} \)

3. \( f(x) = \sin(x) \)  \( \Rightarrow \) \[ f^n(x) = \sin(x) \] \( \Rightarrow \) \[ f^n(0) = 1 \] ; \( P_5(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} \)

4. \( f(x) = \ln(1+x) \)  \( \Rightarrow \) \[ f^n(x) = \ln(1+x) \] \( \Rightarrow \) \[ f^n(0) = 0 \] ; \( P_5(x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} \)

Give the 5th degree Taylor polynomial centered at 0 for each of \( e^x \), \( \cos(x) \), \( \sin(x) \), \( 1 + \frac{1}{1-x} \) and \( \ln(x+1) \).

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