Test 4 Study Materials: A video review and additional practice problems have been posted.

7	8 **EMCF32 due at 9am** Notes: **page**, **4-per**, **video notes**, **video** Homework 11 due in lab/workshop	9 **Extra Series Video**	10 **EMCF33 due at 9am** Notes: **page**, **4-per**, **video notes**, **video** **Homework 12 posted**	11 **Video explanation of why a conditionally convergent series can sum to any value, and an Excel demonstration.**	12 **EMCF34 due at 9am** Blank Slides: **page**, **4-per** Quiz in lab/workshop **Practice Test 4 Opens**	13 **Quiz 11 closes** (10.6-10.7) **2012 Online Test 4 Video Review** **Slides**
14 **More Test 4 Practice Problems** **Solutions**	15 **EMCF35 due at 9am** Homework 12 due in lab/workshop	16	17 **EMCF36 due at 9am** Homework 13 posted	18	19 EMCF37 due at 9am Quiz in lab/workshop	20 **Quiz 12 closes** (11.1-11.4)
21	22 EMCF38 due at 9am Homework 13 due in lab/workshop	**23**	24 EMCF39 due at 9am Homework 14 posted	25	26 EMCF40 due at 9am Quiz in lab/workshop	27 **Quiz 13 closes** (11.5-11.6) **Test 4 starts**

Last day

Popper 27 (0) Converges absolutely, (1) Converges conditionally, (2) Diverges

1. $\sum_{k=0}^{\infty} \frac{(-1)^k}{(k+1)(k+2)}$ 0

2. $\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)}$ 1

3. $\sum_{k=0}^{\infty} \frac{k+1}{3^k}$ 0

4. $\sum_{k=0}^{\infty} \frac{(-1)^k(100)^k}{k!}$ 0

5. $\sum_{k=0}^{\infty} \frac{(-1)^k}{\sqrt{(k+1)(k+2)}}$ 1

6. $\sum_{k=0}^{\infty} \frac{(-1)^k}{\sqrt{k^3+1}}$ 0

7. $\sum_{k=1}^{\infty} k\left(\frac{3}{4}\right)^k$ 0

$\sum \frac{k}{(4/3)^k}$

8. $\sum_{k=0}^{\infty} \frac{2^k+k^4}{3^k}$ 0

9. $\sum (-1)^k \frac{k}{2^k}$. 0

New

Taylor Polynomial Approximations

MOST functions have graphs
that *locally* look like polynomials.

Question: Can we approximate a function if we know the function *completely* at a single point?

??

i.e. you know the function value and all derivative values at this point.

ex. ① $f(x) = e^x$ at $x=0$

② $g(x) = \sin(x)$ at $x=0$

③ $h(x) = \cos(x)$ at $x=0$

Analytic

④ $G(x) = \frac{1}{x}$ at $x=1$

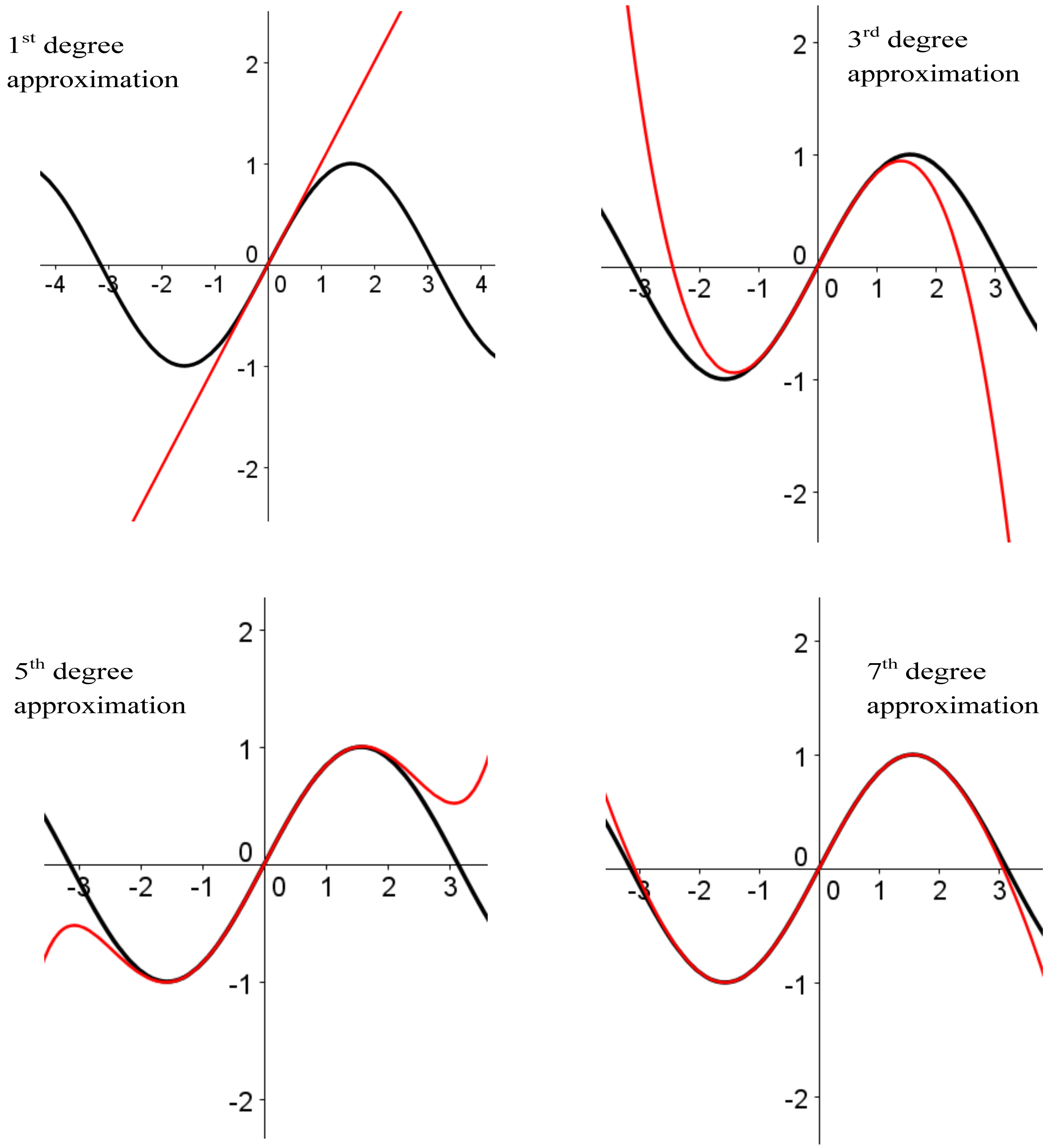
⋮

Question: Can you think of some functions that we know extremely well at a single point?

Done

Goal: Given a function f, and a value $x = a$ where we know f and its derivatives, give a polynomial that approximates f.

Some Geogebra Plots of $\sin(x)$ Versus *Taylor* Polynomial Approximations Centered at $x = 0$.



The Main Idea of Taylor Polynomials: Given a function f, and a value $\boxed{x = a}$, where we know f and its derivatives, give a polynomial that approximates f.

How: Suppose $p(x)$ is a polynomial of degree n.

We want:

$$p(a) = f(a)$$

$$p'(a) = f'(a) \quad (*)$$

$$\vdots$$

$$p^{(n)}(a) = f^{(n)}(a)$$

Write

$$p(x) = \underbrace{c_0}_{= f(a)} + \underbrace{c_1}_{f'(a)}(x-a) + c_2(x-a)^2 + \cdots + c_n(x-a)^n$$

$$f(a) = p(a) = c_0$$

$$p'(x) = c_1 + 2c_2(x-a) + 3c_3(x-a)^2 + \cdots$$

$$f'(a) = p'(a) = c_1$$

$$p''(x) = 2c_2 + 6c_3(x-a) + \cdots$$

$$f''(a) = p''(a) = 2c_2 \Rightarrow c_2 = \frac{1}{2} f''(a)$$

in general ... $c_k = \dfrac{1}{k!} f^{(k)}(a)$

The general process...

Given $f(x)$, a value $x = a$, and a positive integer n, find an n^{th} degree polynomial $p_n(x)$ so that

$$p_n(a) = f(a),\ p_n'(a) = f'(a),\ \cdots,\ p_n^{(n)}(a) = f^{(n)}(a)$$

The Taylor polynomial approximation of f of degree n centered at $x = a$ is given by

$$p_n(x) = f(a) + \sum_{k=1}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k$$

$$f(a) + \frac{f'(a)}{1}(x-a) + \frac{f''(a)}{2}(x-a)^2 + \frac{f'''(a)}{6}(x-a)^3 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n$$

Examples:

Give the 5th degree Taylor polynomial centered at 0 for each of e^x, $\cos(x)$, $\sin(x)$, $\frac{1}{1-x}$ and $\ln(x+1)$.

1. $f(x) = e^x$.   Need $f(0), f'(0), f''(0), f'''(0), f^{(4)}(0), f^{(5)}(0)$.

$f'(x) = e^x,\ f''(x) = e^x,\ \ldots,\ f^{(5)}(x) = e^x$

$\therefore\ f(0) = 1,\ f'(0) = 1,\ \ldots,\ f^{(5)}(0) = 1.$

$$P_5(x) = \underbrace{\underbrace{\underbrace{\underbrace{\underbrace{1 + x}_{= P_1(x)} + \frac{1}{2}x^2}_{= P_2(x)} + \frac{1}{6}x^3}_{= P_3(x)} + \frac{1}{24}x^4}_{= P_4(x)} + \frac{1}{120}x^5}_{= P_5(x)}$$

2. $f(x) = \cos(x)$.

$f(0) = 1$, $f'(x) = -\sin(x)$

$f'(0) = 0$, $f''(x) = -\cos(x)$

$f''(0) = -1$, $f'''(x) = \sin(x)$

$f'''(0) = 0$, $f^{(4)}(x) = \cos(x)$

$f^{(4)}(0) = 1$, $f^{(5)}(x) = -\sin(x)$

$f^{(5)}(0) = 0$

$$P_5(x) = 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4$$

Note: $P_4(x) = P_5(x)$.

3. $f(x) = \sin(x)$. similar

$$P_5(x) = x - \frac{x^3}{6} + \frac{x^5}{120}$$

Give the 5th degree Taylor polynomial centered at 0 for

$$\frac{1}{1-x}$$

$f(x) = \frac{1}{1-x}$. $f(0) = 1$

$f'(x) = \frac{1}{(1-x)^2}$ $f'(0) = 1$

$f''(x) = \frac{2}{(1-x)^3}$ $f''(0) = 2$

$f'''(x) = \frac{3!}{(1-x)^4}$ $f'''(0) = 3!$

$f^{(4)}(x) = \frac{4!}{(1-x)^5}$ $f^{(4)}(0) = 4!$

$f^{(5)}(x) = \frac{5!}{(1-x)^6}$ $f^{(5)}(0) = 5!$

$\underline{\text{Aside}}$: $\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k$, $|x|<1$

$= \underline{1 + x + x^2 + x^3 + x^4 + x^5} + \cdots$

$\therefore$ $p_5(x) = 1 + 1 \cdot x + \frac{2}{2}x^2 + \frac{3!}{3!}x^3 + \frac{4!}{4!}x^4 + \frac{5!}{5!}x^5$

$= 1 + x + x^2 + x^3 + x^4 + x^5$.