Information

Test 4 is approaching.

A **video review** is posted, along with
additional review problems.

Review:

What is the formula for the n^{th} degree Taylor polynomial approximation of a function $f(x)$ centered at $x = a$?

$$f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n$$

$p_{0,a}(x)$, $p_{1,a}(x)$, $p_{2,a}(x)$

Notation: $p_{n,a}(x)$ or $p_n(x)$.

Example: Give the Taylor polynomials of degree 7 centered at 0 for each of $\sin(x)$, $\cos(x)$, $\exp(x)$ and $1/(1-x)$.

$f(x) = \sin(x)$ $f(0) = 0$

$f'(x) = \cos(x)$ $f'(0) = 1$

$f''(x) = -\sin(x)$ $f''(0) = 0$

$f'''(x) = -\cos(x)$ $f'''(0) = -1$

$f^{(4)}(x) = \sin(x)$ $f^{(4)}(0) = 0$

$f^{(5)}(x) = \cos(x)$ $f^{(5)}(0) = 1$

$f^{(6)}(x) = -\sin(x)$ $f^{(6)}(0) = 0$

$f^{(7)}(x) = -\cos(x)$ $f^{(7)}(0) = -1$

$$P_7(x) = 0 + 1\cdot x + \frac{0}{2}x^2 + \frac{(-1)}{6}x^3 + \frac{0}{24}x^4 + \frac{1}{120}x^5 + \frac{0}{720}x^6 + \frac{(-1)}{5040}x^7$$

$$= x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040}$$

7th degree Taylor polynomial approximation for $\sin(x)$ centered at 0.

$f(x) = \cos(x) \qquad f(0) = 1$

$f'(x) = -\sin(x) \qquad f'(0) = 0$

$f''(x) = -\cos(x) \qquad f''(0) = -1$

$f'''(x) = \sin(x) \qquad f'''(0) = 0$

$f^{(4)}(x) = \cos(x) \qquad f^{(4)}(0) = 1$

$f^{(5)}(x) = -\sin(x) \qquad f^{(5)}(0) = 0$

$f^{(6)}(x) = -\cos(x) \qquad f^{(6)}(0) = -1$

$f^{(7)}(x) = \sin(x) \qquad f^{(7)}(0) = 0$

$$P_7(x) = 1 + 0 \cdot x + \frac{(-1)}{2}x^2 + \frac{0}{6} \cdot x^3 + \frac{1}{24}x^4 + \frac{0}{120}x^5 + \frac{-1}{720}x^6 + \frac{0}{5040}x^7$$

$$= 1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720}$$

7th degree Taylor polynomial approximation for $\cos(x)$ centered at 0.

$$f(x) = e^x \qquad f(0) = 1$$

$$f'(x) = e^x \qquad f'(0) = 1$$

$$\vdots \qquad\qquad \vdots$$

$$f^{(7)}(x) = e^x \qquad f^{(7)}(0) = 1$$

$$P_7(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \frac{x^6}{720} + \frac{x^7}{5040}$$

7th degree Taylor polynomial approximation for $\exp(x)$ centered at 0.

Example: Give the Taylor polynomial of degree 7 centered at 0 for $f(x) = e^{2x}$.

$f(x) = e^{2x}$ $\quad$ $f(0) = 1$

$f'(x) = 2e^{2x}$ $\quad$ $f'(0) = 2$

$\vdots$ $\quad$ $\vdots$

$f^{(n)}(x) = 2^n e^{2x}$ $\quad$ $f^{(n)}(0) = 2^n$

$$P_7(x) = 1 + 2x + \frac{4}{2}x^2 + \frac{8}{6}x^3 + \frac{16}{24}x^4 + \frac{32}{120}x^5 + \frac{64}{720}x^6 + \frac{128}{5040}x^7$$

7th degree Taylor polynomial approximation for $\exp(2x)$ centered at 0.

Example: Give the Taylor polynomial of degree 4 centered at 1 for $f(x) = \ln(x)$.

$f(x) = \ln(x) \qquad f(1) = 0$

$f'(x) = \frac{1}{x} \qquad f'(1) = 1$

$f''(x) = \frac{-1}{x^2} \qquad f''(1) = -1$

$f'''(x) = \frac{2}{x^3} \qquad f'''(1) = 2$

$f^{(4)}(x) = \frac{-6}{x^4} \qquad f^{(4)}(1) = -6$

$P_{4,1}(x) = 0 + 1 \cdot (x-1) + \frac{-1}{2}(x-1)^2 + \frac{2}{6}(x-1)^3 + \frac{-6}{24}(x-1)^4$

$P_{4,1}(x) = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4$

Example: Give the Taylor polynomial of degree 4 centered at 0 for $f(x) = x\sin(x) + 3x^2 - 2x + 1$.

$f(0) = 1$

$f'(x) = x\cos(x) + \sin(x) + 6x - 2$ $\qquad f'(0) = -2$

$f''(x) = -x\sin(x) + 2\cos(x) + 6$ $\qquad f''(0) = 8$

$f'''(x) = -x\cos(x) - 3\sin(x)$ $\qquad f'''(0) = 0$

$f^{(4)}(x) = x\sin(x) - 4\cos(x)$ $\qquad f^{(4)}(0) = -4$

$$P_4(x) = 1 - 2x + \frac{8}{2}x^2 + 0x^3 - \frac{4}{24}x^4$$

$$= 1 - 2x + 4x^2 - \frac{1}{6}x^4$$

Interesting note: we want 4th degree

$f(x) = x\sin(x) + 3x^2 - 2x + 1$

ditto

2nd degree poly centered at 0.

3rd degree approx $x - \frac{x^3}{6}$

then $x\left(x - \frac{x^3}{6}\right) + 3x^2 - 2x + 1$

$$1 - 2x + 4x^2 - \frac{x^4}{6}$$

11 Questions, 20 seconds each...

Popper 28

1. $$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\sqrt{n}}{n+3}$$

(0) Converges Absolutely
(1) Converges Conditionally
(2) Diverges

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2. $$\sum_{n=1}^{\infty} \frac{\cos(\pi n)}{n^2}$$

(0) Converges Absolutely
(1) Converges Conditionally
(2) Diverges

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3. $\sum_{n=0}^{\infty}\left(\frac{(-1)^n 3^n}{4^n + 3n}\right)$

(0) Converges Absolutely
(1) Converges Conditionally
(2) Diverges

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4. $$\sum_{n=2}^{\infty} \frac{(-1)^n 3^n}{n!}$$

(0) Converges Absolutely
(1) Converges Conditionally
(2) Diverges

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5. $$\sum_{n=0}^{\infty} \frac{(-1)^n 10n^2}{3^n}$$

(0) Converges Absolutely
(1) Converges Conditionally
(2) Diverges

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6. Give the 4^{th} degree Taylor polynomial centered at $x = 0$ for e^x.

(0) $1 - x + \frac{x^2}{2} - \frac{x^3}{3} + \frac{x^4}{4}$

(1) $1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24}$

(2) $x - \frac{x^3}{6}$

(3) $1 - \frac{x^2}{2} + \frac{x^4}{24}$

(4) None of these.

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7. Give the 4th degree Taylor polynomial centered at $x = 0$ for $\cos(x)$.

(0) $1 - x + \frac{x^2}{2} - \frac{x^3}{3} + \frac{x^4}{4}$

(1) $1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24}$

(2) $x - \frac{x^3}{6}$

(3) $1 - \frac{x^2}{2} + \frac{x^4}{24}$

(4) None of these.

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8. Give the 4th degree Taylor polynomial centered at $x = 0$ for $\sin(x)$.

(0) $1 - x + \frac{x^2}{2} - \frac{x^3}{3} + \frac{x^4}{4}$

(1) $1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24}$

(2) $x - \frac{x^3}{6}$

(3) $1 - \frac{x^2}{2} + \frac{x^4}{24}$

(4) None of these.

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9. $$\sum_{n=2}^{\infty} \frac{(-1)^n}{3n+2}$$

(0) Converges Absolutely
(1) Converges Conditionally
(2) Diverges

Popper 28

10. $$\sum_{n=0}^{\infty} \frac{(-1)^n 10n^2}{3^n}$$

(0) Converges Absolutely
(1) Converges Conditionally
(2) Diverges

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11. The function f has values given by $f(0) = 1$, $f'(0) = -1$, $f''(0) = 0$, $f'''(0) = 3$ and $f^{(4)}(0) = -2$. Give $p_{4,0}(x)$.

(0) There is not enough information.

(1) $1 - x + 3x^3 - 2x^4$

(2) $1 - x + x^3 - \frac{1}{2}x^4$

(3) $1 - x + \frac{1}{2}x^3 - \frac{1}{12}x^4$

(4) None of these.