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14 **More Test 4 Practice Problems** **Solutions**	15 **EMCF35 due at 9am** Notes: **page**, **4-per**, **video notes**, **video** Homework 12 due in lab/workshop	16	17 **EMCF36 due at 9am** Notes: **page**, **4-per** **Homework 13 posted**	18 **Video: Taylor Error Estimate Example**	19 **EMCF37 due at 9am** Blank Slides: **page** **4-per** Quiz in lab/workshop	20 **Quiz 12 closes** (11.1-11.4) **2012 Online Test 4 Video Review** **Slides**
21 **Note: Homework 13 is not due until the 29th!!**	22 **EMCF38 due at 9am**	**23** **Complete the Online Teacher Evaluation by May 3 to Receive 5 Bonus Points**	24 EMCF39 due at 9am	25	26 EMCF40 due at 9am Quiz in lab/workshop	27 **Quiz 13 closes** (11.5-11.6) **Test 4 starts**
28	29 EMCF41 due at 9am Homework 13 due in lab/workshop **Last day of class**	30 **Practice Test 4 Closes**	May 1	2	3	4 **Quiz 14 closes** (11.7-11.8)

Popper 30

1. Give the 3rd degree Taylor polynomial for sin($2x$) centered at 0.

(0) $x - \frac{1}{162}x^3$

(1) $1 - 2x^2$

(2) $1 - \frac{1}{18}x^2$

(3) $2x - \frac{8}{3}x^3$

(4) $1 - \frac{1}{2}x + \frac{1}{8}x^2 - \frac{1}{48}x^3$

(5) None of these.

Popper 30

2. Give the 3rd degree Taylor polynomial for $\cos(x/3)$ centered at 0.

(0) $x - \frac{1}{162}x^3$

(1) $1 - 2x^2$

(2) $1 - \frac{1}{18}x^2$

(3) $2x - \frac{8}{3}x^3$

(4) $1 - \frac{1}{2}x + \frac{1}{8}x^2 - \frac{1}{48}x^3$

(5) None of these.

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3. Give the 3rd degree Taylor polynomial for exp(-x/2) centered at 0.

(0) $x - \frac{1}{162}x^3$

(1) $1 - 2x^2$

(2) $1 - \frac{1}{18}x^2$

(3) $2x - \frac{8}{3}x^3$

(4) $1 - \frac{1}{2}x + \frac{1}{8}x^2 - \frac{1}{48}x^3$

(5) None of these.

Taylor Series: (essentially, infinite degree Taylor polynomials)

Power series

Definition: If $f(x)$ is defined and has derivatives of every order at $x = a$, then the Taylor series for $f(x)$ centered at $x = a$ is given by

$$f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots$$

$$= \sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a)^k = \lim_{n \to \infty} P_{n,a}(x)$$

...just like a Taylor polynomial, but the degree is infinite...

power

Example: Give the ~~Taylor~~ series for e^x, $\sin(x)$ and $\cos(x)$ centered at $x = 0$.

$\underline{\sin(x)}$:

$$P_{2n+1}(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots + \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

$$\text{T.S.} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

$\underline{\cos(x)}$:

$$P_{2n}(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots + \frac{(-1)^n x^{2n}}{(2n)!}$$

$$\text{T.S.} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

e^x:

$$P_n(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots + \frac{x^n}{n!}$$

$$\text{T.S.} = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

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4. Give the Taylor series centered at 0 for $\sin(x)$.

(0) $\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} x^{2k}$

(1) $\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} x^{2k+1}$

(2) $\sum_{k=0}^{\infty} \frac{1}{k!} x^k$

(3) $\sum_{k=0}^{\infty} x^k$

(4) None of these.

5. Give the Taylor series centered at 0 for $\cos(x)$.

(0) $\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} x^{2k}$

(1) $\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} x^{2k+1}$

(2) $\sum_{k=0}^{\infty} \frac{1}{k!} x^k$

(3) $\sum_{k=0}^{\infty} x^k$

(4) None of these.

6. Repeat for e^x.

Comment:

$\sin(x)$, $\cos(x)$ and e^x are equal to their Taylor Series centered at 0 for all x.

This is not the case for all functions.

e.g. $1/(1 + x)$, $\ln(1 + x)$, $\arctan(x)$ and many others.

$\ln(x)$ Centered at $x=1$.

$\frac{1}{1-x}$ Centered at $x=0$

Example: Give the Taylor series for

$$f(x) = \frac{1}{1-x} \qquad f(0) = 1$$

$\boxed{\text{centered at } x = 0}$. For which values of x does this series converge?

$$f'(x) = \frac{1}{(1-x)^2} \qquad f'(0) = 1$$

$$f''(x) = \frac{2}{(1-x)^3} \qquad f''(0) = 2$$

$$f'''(x) = \frac{6}{(1-x)^4} \qquad f'''(0) = 6$$

$$\vdots \qquad \vdots$$

$$f^{(k)}(x) = \frac{k!}{(1-x)^{k+1}} \qquad f^{(k)}(0) = k!$$

$$\text{T.S.} = \sum_{k=0}^{\infty} \frac{f^{(k)}(0)}{k!} x^k = \sum_{k=0}^{\infty} x^k$$

Summary: $\sum_{k=0}^{\infty} x^k$ is the T.S. centered at 0 for $f(x) = \frac{1}{1-x}$.

Geometric Series

$= \frac{1}{1-x}$ iff $|x| < 1$.

This series diverges when $|x| \geq 1$.

A power series centered at a has the form

$$\sum_{k=0}^{\infty} b_k (x-a)^k$$

The radius of convergence of a power series is the larges value of R so that the power series converges for $|x-a| < R$.

Notes:

1. Absolute convergence determines the radius of convergence.

2. If a power series is equal to a function on an interval, then the power series is the Taylor series for the function.

3. Power series can be integrated and differentiated in the interior of their interval of convergence, and the power series, the derivative and the antiderivative all have the SAME radius of convergence.

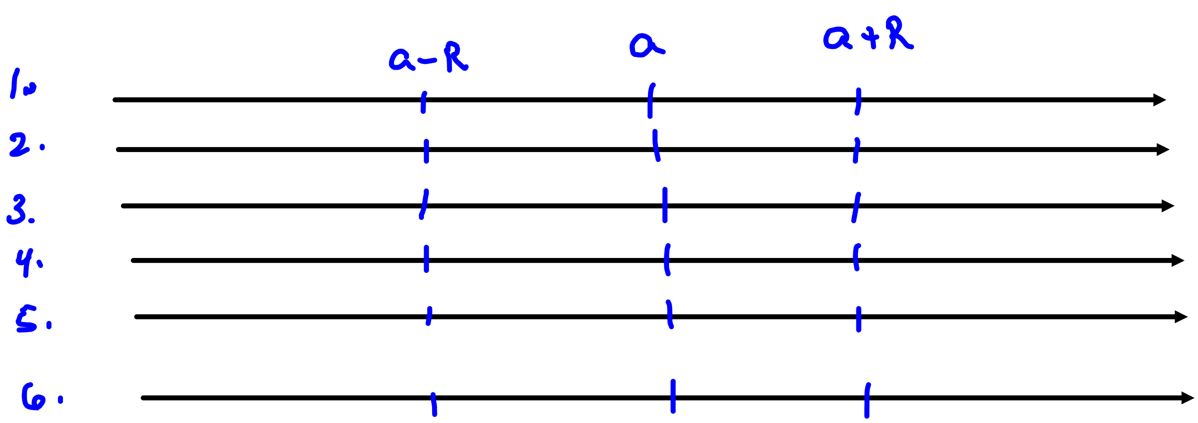
The value(s) of x where

$$\sum_{k=0}^{\infty} b_k (x-a)^k$$

converges will be one of

> **Fact:** Absolute converges determines the radius of convergence.

1. Only $x=a$.
2. $(-\infty, \infty)$
3. $(a-R, a+R)$
4. $[a-R, a+R)$
5. $(a-R, a+R]$
6. $[a-R, a+R]$.



Example: Determine the values of x where

$$\sum_{k=0}^{\infty} \frac{(-1)^k}{k+1}(x-1)^k \quad \text{converges,}$$

and give the radius of convergence.