Information

No Office Hours Today!!

| 25 | EME 230 due at 8am
| 26 | Practice Test 2 from Class
| 28 | 3Q PTMC due at 8am
| 29 | Exam 2 due at 8am
| 30 | Exam 2 due at 8am

Exam 2 due at
Practice Test 2 from Class

Due: Exam 2 due at 8am
Last day of class

\[
\sum_{k=0}^\infty b_k (x-a)^k
\]

The value(s) of x where

\[
\sum_{k=0}^\infty b_k (x-a)^k
\]

converges will be one of

1. Only \( x = a \)
2. \((-\infty, \infty)\)
3. \((a-R, a+R)\)
4. \([a-R, a+R)\)
5. \((a-R, a+R]\)

\[ R > a - R \]

\[ R < a + R \]

\[ a - R \]

\[ a + R \]

The radius of convergence of a power series is the largest value of \( R \) so that the power series converges for \( |x-a| < R \).

Notes:

1. Absolute convergence determines the radius of convergence.
2. If a power series is equal to a function on an interval, then the power series is the Taylor series for the function.
3. Power series can be integrated and differentiated in the interior of their interval of convergence, and the power series, the derivative and the antiderivative all have the same radius of convergence.

Example: Determine the values of x where

\[
\frac{k!}{x^k}
\]

converges, and give the radius of convergence.

1. Only \( x = a \)
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**Important Fact:** If a power series centered at \( x = a \) has a radius of convergence \( R > 0 \), then the power series can be differentiated and integrated on \((a - R, a + R)\), and the new series will converge on \((a - R, a + R)\), and maybe at the endpoints.

Example: Find the interval and radius of convergence for \( f(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!} \). Then give the derivatives \( f^{(n)}(a) \) at \( x = 0 \).

\[
\text{Find } f^{(n)}(0) \text{ so } \lim_{n \to \infty} f^{(n)}(0) = f(x) = \sqrt[3]{x}
\]

\[
\text{Integrate (sum by } \text{term)}
\]

\[
F(x) = \sum_{n=0}^{\infty} \frac{x^{n+1}}{(n+1)!} + C
\]

\[
F(0) = C
\]

\[
\Rightarrow F(x) = \sum_{n=0}^{\infty} \frac{x^{n+1}}{(n+1)!} + 1
\]

See the video and video notes for the remainder of this problem.

Example: Let \( f(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!} \). Give \( f^{(n)}(0) \).

See the video and video notes for the remainder of this problem.
Example: Give the Taylor series centered at 0 for \( \frac{1}{1-x} \),

\[ \frac{1}{1+x}, \ln(1+x), \ln(1+x^2), \text{ and } x^2 \ln(1+x^2). \]

In each case, give the radius of convergence.

Next Time