Information

No Office Hours Today!! Later today.

21 **Note: Homework 13 is not due until the 29th!!** **Please consider taking the time to complete this survey.**	22 **EMCF38 due at 9am** Blank Slides: **page, 4-per, video notes, video**	23 **Complete the Online Teacher Evaluation by May 3 to Receive 5 Bonus Points**	24 EMCF39 due at 9am	25	26 EMCF40 due at 9am Quiz in lab/workshop	27 **Quiz 13 closes** (11.5-11.6) **Test 4 starts**
28	29 EMCF41 due at 9am Homework 13 due in lab/workshop **Last day of class**	30 **Practice Test 4 Closes**	May 1	2	3	4 **Quiz 14 closes** (11.7-11.8)
5	6 **Final Exam Starts**	7	8	9	10	11

A power series centered at a has the form

$$\sum_{k=0}^{\infty} b_k (x-a)^k$$

Taylor series

If this P.S. is $f(x)$ then $b_k = \dfrac{f^{(k)}(a)}{k!}$

The radius of convergence of a power series is the largest value of R so that the power series converges for $|x-a| < R$.

? ?

$a-R$ a $a+R$

R R

Notes:

1. Absolute convergence determines the radius of convergence.

2. If a power series is equal to a function on an interval, then the power series is the Taylor series for the function.

3. Power series can be integrated and differentiated in the interior of their interval of convergence, and the power series, the derivative and the antiderivative all have the SAME radius of convergence.

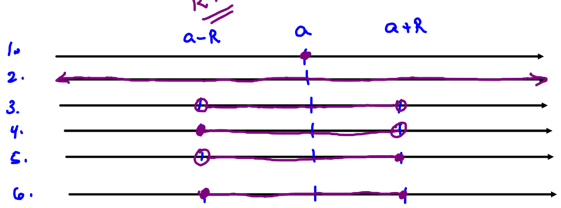
The value(s) of x where

$$\sum_{k=0}^{\infty} b_k (x-a)^k$$

converges will be one of

Fact: Absolute convergence determines the radius of convergence.

1. Only $x = a$. $R = 0$
2. $(-\infty, \infty)$ $R = \infty$
3. $(a-R, a+R)$ $R > 0$
4. $[a-R, a+R)$
5. $(a-R, a+R]$
6. $[a-R, a+R]$.



Example: Determine the values of x where

$$\sum_{k=0}^{\infty} \frac{(-1)^k}{k+1}(x-1)^k \text{ converges,}$$

and give the radius of convergence.

PS $\Leftrightarrow$ centered at $x=1$.

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① Get R. Abs. conv. determines the radius of conv.

$$\sum_{k=0}^{\infty} \left| \frac{(-1)^k}{k+1}(x-1)^k \right| = \sum_{k=0}^{\infty} \frac{|x-1|^k}{k+1}$$

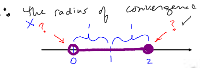
$\boxed{\text{ratio test}}$: $\lim_{k\to\infty} \dfrac{\frac{|x-1|^{k+1}}{k+2}}{\frac{|x-1|^k}{k+1}} = \lim_{k\to\infty} \dfrac{(k+1)|x-1|^{k+1}}{(k+2)|x-1|^k}$ $\xrightarrow{} 1$

$= |x-1|$

$\therefore$ Conv. for $|x - 1| < 1$

a R

$\therefore$ the radius of convergence is 1.



$x=0$: Substitute $x=0$ into $\sum_{k=0}^{\infty} \frac{(-1)^k}{k+1}(x-1)^k$,

$$\sum_{k=0}^{\infty} \frac{(-1)^k}{k+1}(-1)^k = \sum_{k=0}^{\infty} \frac{1}{k+1}$$

Diverges by CT with $\sum \frac{1}{k}$

$x=2$: Substitute $x=2$ into $\sum_{k=0}^{\infty} \frac{(-1)^k}{k+1}(x-1)^k$,

$$\sum_{k=0}^{\infty} \frac{(-1)^k}{k+1} = \sum_{k=0}^{\infty} (-1)^k \cdot \frac{1}{k+1}$$

Alternating series : Note - ① $\frac{1}{k+1} \geq 0$

② $\frac{1}{k+1} \to 0$

③ $\frac{1}{k+1}$ is decreasing

$\therefore$ by AST, this series converges.

In summary: Radius of convergence $= 1$.

The series converges on $(0, 2]$.

$\boxed{\text{i.e. for } 0 < x \leq 2}$

Important Fact: If a power series centered at $x = a$ has a radius of convergence $R > 0$, then the power series can be differentiated and integrated term by term on $(a - R, a + R)$, and the new series will converge on $(a - R, a + R)$, and maybe at the endpoints.

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Example: Find the interval and radius of convergence for $f(x) = \sum_{n=0}^{\infty} \frac{x^n}{n^2+1}$. Then give the antiderivative $F(x)$ of this power series that satisfies $F(0) = 2$, and find $f'(x)$. Finally, give the radius and intervals of convergence for each of $F(x)$ and $f'(x)$. } vid

P.S. (T.S. for $f(x)$ centered at $x=0$

Aside: what is $f^{(12)}(0)$?

Coef. in front of x^{12} is $\frac{1}{12^2+1} = \frac{1}{145}$

$\therefore \frac{1}{145} = \frac{f^{(12)}(0)}{12!}$

$\Rightarrow f^{(12)}(0) = \frac{12!}{145}$.

$\sum_{n=0}^{\infty} \frac{x^n}{n^2+1}$: Radius of conv. ?

Use abs. conv.: $\sum_{n=0}^{\infty} \left| \frac{x^n}{n^2+1} \right|$

$= \sum_{n=0}^{\infty} \frac{|x|^n}{n^2+1}$

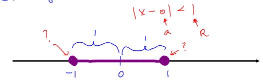
Ratio Test: $\lim_{n\to\infty} \frac{\frac{|x|^{n+1}}{(n+1)^2+1}}{\frac{|x|^n}{n^2+1}} = \lim_{n\to\infty} \frac{(n^2+1)|x|^{n+1}}{((n+1)^2+1)|x|^n}$

$= \lim_{n\to\infty} \boxed{\frac{(n^2+1)}{n^2+2n+2}} |x|$ → 1

$= |x|$

Convergence for $|x| < 1$.

$|x - 0| < 1$ (a, R)



Check $x = -1$:
$x = 1$: Plug in.

You will find convergence for both

Summary: $\sum_{n=0}^{\infty} \frac{x^n}{n^2+1}$ has radius of conv. $= 1$ and it converges on $[-1, 1]$.

$$f(x) = \sum_{n=0}^{\infty} \frac{x^n}{n^2+1} = \sum_{n=0}^{\infty} \frac{1}{n^2+1} x^n$$

Find $F(x)$ so that $F'(x) = f(x)$ and $\boxed{F(0) = 2}$

Integrate (term by term)

$$\left[\sum_{n=0}^{\infty} \frac{1}{n^2+1} \cdot \frac{x^{n+1}}{n+1}\right] + C$$

$$F(x) = \left[\sum_{n=0}^{\infty} \frac{x^{n+1}}{(n^2+1)(n+1)}\right] + C$$

$F(0) = 2$:

$$2 = \left[\sum_{n=0}^{\infty} \frac{0^{n+1}}{(n^2+1)(n+1)}\right] + C$$

$\therefore C = 2$

$$\Rightarrow F(x) = \left[\sum_{n=0}^{\infty} \frac{x^{n+1}}{(n^2+1)(n+1)}\right] + 2$$

See the video and video notes for the remainder of this problem.

Example: Let $f(x) = \sum_{n=0}^{\infty} \frac{x^n}{n^2+1}$. Give $f^{(9)}(0)$.

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Example: Give the Taylor series centered at 0 for $\frac{1}{1-x}$, $\frac{1}{1+x}$, $\ln(1+x)$, $\ln\left(1+x^3\right)$, and $x^2\ln\left(1+x^3\right)$. In each case, give the radius of convergence.

Next Time