Office Hours Today: 1:30 - 2:30

14 More Test 4 Practice Problems Solutions	15 EMCF35 due at 9am Notes: page, 4-per, video notes, video Homework 12 due in lab/workshop	16	17 EMCF36 due at 9am Notes: page, 4-per Homework 13 posted	18 Video: Taylor Error Estimate Example	19 EMCF37 due at 9am Notes: page, 4-per, video notes, video Quiz in lab/workshop	20 Quiz 12 closes (11.1-11.4) 2012 Online Test 4 Video Review Slides
21 Note: Homework 13 is not due until the 29th!! Please consider taking the time to complete this survey.	22 EMCF38 due at 9am Blank Slides: page, 4-per, video notes, video	23 Complete the Online Teacher Evaluation by May 3 to Receive 5 Bonus Points	24 EMCF39 due at 9am Blank Slides: page, 4-per	25	26 EMCF40 due at 9am Quiz in lab/workshop	27 Quiz 13 closes (11.5-11.6) Test 4 starts
28	29 EMCF41 due at 9am Homework 13 due in lab/workshop Last day of class	30 Practice Test 4 Closes	May 1	2	3	4 Quiz 14 closes (11.7-11.8)
5	6 Final Exam Starts	7	8	9	10	11

Apr 24-9:50 AM

Recall: If a power series centered at $x = a$ has a radius of convergence $R > 0$, then the power series can be differentiated and integrated term by term, and R will also be the radius of convergence of the new series.

Example: Give the Taylor series centered at 0 for $\frac{1}{1-x}$, $\frac{1}{1+x}$, $\ln(1+x)$, $\ln(1+x^3)$, and $x^2\ln(1+x^3)$.
In each case, give the radius of convergence.

$\frac{1}{1+x^2}$, $\arctan(x)$,

Recall: $\sum_{n=0}^{\infty} r^n = \begin{cases} \frac{1}{1-r} & \text{if } |r|<1 \\ \text{diverges} & \text{otherwise} \end{cases}$

$\therefore \quad \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \quad \text{for } |x|<1$

$$\frac{1}{1+x} = \frac{1}{1-(-x)} = \sum_{n=0}^{\infty}(-x)^n, \quad |x|<1$$
$$= \sum_{n=0}^{\infty}(-1)^n x^n.$$

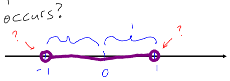
$$\frac{1}{1+x^2} = \frac{1}{1-(-x^2)} = \sum_{n=0}^{\infty}(-x^2)^n, \quad |-x^2|<1 \text{ i.e. } |x|<1$$
$$= \sum_{n=0}^{\infty}(-1)^n x^{2n}$$

radius of conv. $= 1$

$$\arctan(x) + C = \int \frac{1}{1+x^2}\,dx$$
$$= \int \sum_{n=0}^{\infty}(-1)^n x^{2n}\,dx$$
$$= \sum_{n=0}^{\infty}\int (-1)^n x^{2n}\,dx$$
$$= \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{2n+1}$$

$\Rightarrow$ radius of conv. is 1.

Values of x where conv. occurs?



$\therefore$

$$\arctan(x) + C = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{2n+1}$$

for $-1 \le x \le 1$

C? Subst. $x = 0$

$0 + C = 0$

$\therefore \quad \arctan(x) = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{2n+1}$ for $-1 \le x \le 1$.

This P.S. is the Taylor series centered at 0.

$$\left.\frac{d^6 \arctan(x)}{dx^6}\right|_{x=0} = ?$$

shows up as part of the coef on x^6

Full coef: $\frac{f^{(6)}(0)}{6!}$

Note: There are no even powers in this P.S.

$\therefore$ Coef on x^6 is 0

$$\Rightarrow \left.\frac{d^6 \arctan(x)}{dx^6}\right|_{x=0} = 0$$

hm? What is $\boxed{\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}}$

$$\left.\frac{d^7 \arctan(x)}{dx^7}\right|_{x=0} = ? \quad = -720$$

Coef on x^7? use $n=3$

$$\text{Coef} = \frac{(-1)^3}{7} = \frac{f^{(7)}(0)}{7!}$$

$$\Rightarrow f^{(7)}(0) = \frac{(-1)\cdot 7!}{7} = -6! = -720$$

$x=-1$: Subst. into P.S.

$$\sum_{n=0}^{\infty} (-1)^n \cdot \frac{-1}{2n+1} = \sum_{n=0}^{\infty} (-1)^{n+1} \underbrace{\frac{1}{2n+1}}_{a_n}$$

Alt. Series

$\sum (-1)^{n+1} a_n$

Alt. series test: $a_n \geq 0$

$a_n \to 0$ as $n \to \infty$

a_n decreasing

Our case: $\frac{1}{2n+1} \geq 0$ ✓

$\frac{1}{2n+1} \to 0$ as $n \to \infty$ ✓

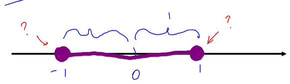
$\frac{1}{2n+1}$ is decreasing ✓

∴ AST ⇒ the series converges.

$x=1$: Subst. into P.S.

$$\sum_{n=0}^{\infty} (-1)^n \frac{1}{2n+1}$$

Similar. Converges by AST.



∴ our P.S. for $\arctan(x)$ converges on $[-1, 1]$.

$$\frac{1}{1+x} = \frac{1}{1-(-x)} = \sum_{n=0}^{\infty} (-x)^n, \quad |x|<1$$

$$= \sum_{n=0}^{\infty} (-1)^n x^n. \quad \text{radius of conv.} = 1$$

$\ln(1+x)$:

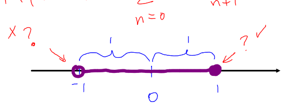
$$\ln(1+x) + C = \int \frac{1}{1+x}\,dx = \int \sum_{n=0}^{\infty} (-1)^n x^n\,dx = \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1}$$

radius of conv. is 1.

$C=?$ Subst $x=0$.

$0 + C = 0$

∴ $\ln(1+x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1}$



You do it: $x=-1$ diverges

$x=1$ converges.

∴ $\ln(1+x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1}$ for $-1 < x \leq 1$.

∴ $\ln(1+u) = \sum_{n=0}^{\infty} (-1)^n \frac{u^{n+1}}{n+1}$ for $-1 < u \leq 1$.

$\ln(1+\underbrace{x^3}_{u})$:

$$= \sum_{n=0}^{\infty} (-1)^n \frac{(x^3)^{n+1}}{n+1}, \quad \text{for } -1 < x^3 \leq 1$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{x^{3n+3}}{n+1}, \quad \text{for } -1 < x \leq 1.$$

$$\underbrace{\ln(1+x^3)} = \underbrace{\sum_{n=0}^{\infty} (-1)^n \frac{x^{3n+3}}{n+1}},$$

for $-1 < x \leq 1.$

$$x^2 \underbrace{\ln(1+x^3)} = x^2 \cdot \sum_{n=0}^{\infty} (-1)^n \frac{x^{3n+3}}{n+1}$$

for $-1 < x \leq 1.$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{x^{3n+5}}{n+1}$$

for $-1 < x \leq 1.$