Office Hours Today: 1:30 - 2:30

14 More Test 4 Practice Problems Solutions	15 EMCF35 due at 9am Notes: page, 4-per, video notes, video Homework 12 due in lab/workshop	16	17 EMCF36 due at 9am Notes: page, 4-per Homework 13 posted	18 Video: Taylor Error Estimate Example	19 EMCF37 due at 9am Notes: page, 4-per, video notes, video Quiz in lab/workshop	20 Quiz 12 closes (11.1-11.4) 2012 Online Test 4 Video Review Slides
21 **Note: Homework 13 is not due until the 29th!!** Please consider taking the time to complete this survey.	22 EMCF38 due at 9am Blank Slides: page, 4-per, video notes, video	**23** Complete the Online Teacher Evaluation by May 3 to Receive 5 Bonus Points	24 EMCF39 due at 9am Blank Slides: page, 4-per	25	26 EMCF40 due at 9am Quiz in lab/workshop	27 Quiz 13 closes (11.5-11.6) **Test 4 starts**
28	29 EMCF41 due at 9am Homework 13 due in lab/workshop **Last day of class**	30 Practice Test 4 Closes	May 1	2	3	4 Quiz 14 closes (11.7-11.8)
5	6 **Final Exam Starts**	7	8	9	10	11

Recall: If a power series centered at $x = a$ has a radius of convergence $R > 0$, then the power series can be differentiated and integrated term by term, and R will also be the radius of convergence of the new series.

Example: Give the Taylor series centered at 0 for $\frac{1}{1-x}$, $\frac{1}{1+x}$, $\ln(1+x)$, $\ln\left(1+x^3\right)$, and $x^2\ln\left(1+x^3\right)$.

In each case, give the radius of convergence.

$\frac{1}{1+x^2}$, $\arctan(x)$,

Recall: $\displaystyle\sum_{n=0}^{\infty} r^n = \begin{cases} \frac{1}{1-r} & \text{if } |r|<1 \\ \text{diverges} & \text{otherwise} \end{cases}$

$$\therefore \quad \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \quad \text{for } |x|<1$$

$$\frac{1}{1+x} = \frac{1}{1-(-x)} = \sum_{n=0}^{\infty} (-x)^n, \quad |x|<1$$

$$= \sum_{n=0}^{\infty} (-1)^n x^n.$$

$$\frac{1}{1+x^2} = \frac{1}{1-(-x^2)} = \sum_{n=0}^{\infty} (-x^2)^n, \quad |-x^2|<1$$

$$= \sum_{n=0}^{\infty} (-1)^n x^{2n} \quad \text{i.e. } |x|<1$$

radius of conv. = 1

$$\arctan(x) + C = \int \frac{1}{1+x^2}\,dx$$

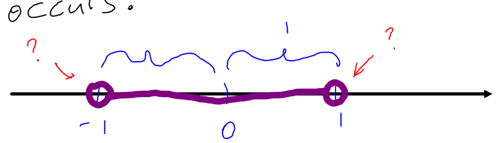
$$= \int \sum_{n=0}^{\infty} (-1)^n x^{2n}\,dx$$

$$= \sum_{n=0}^{\infty} \int (-1)^n x^{2n}\,dx$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$$

$\Rightarrow$ radius of conv. is 1.

Values of x where conv. occurs?



$\therefore$

$$\arctan(x) + C = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$$

for $-1 \le x \le 1$

$\underline{\underline{C}}$? Subst. $x = 0$

$$0 + C = 0$$

$\therefore$ $$\arctan(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$$

This P.S. is the Taylor series centered at 0.

for $\underline{\underline{-1 \le x \le 1}}$.

$$\left.\frac{d^6 \arctan(x)}{dx^6}\right|_{x=0} = \;?$$

← shows up as part of the coef on x^6

$\underline{\text{Full coef:}}$

$$\frac{f^{(6)}(0)}{6!}$$

$\underline{\text{Note}}$: There are no even powers in this PS.

$\therefore$ Coef on x^6 is 0

$$\Rightarrow \left.\frac{d^6 \arctan(x)}{dx^6}\right|_{x=0} = 0$$

hm? What is $\boxed{\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}}$

$$\left.\frac{d^7 \arctan(x)}{dx^7}\right|_{x=0} = \,? = -720$$

Coef on x^7? use $n=3$

$$\text{Coef} = \frac{(-1)^3}{7} = \frac{f^{(7)}(0)}{7!}$$

$$\Rightarrow f^{(7)}(0) = \frac{(-1)\cdot 7!}{7}$$

$$= -6!$$

$$= -720$$

$x = -1$: Subst. into P.S.

$$\sum_{n=0}^{\infty} (-1)^n \cdot \frac{-1}{2n+1}$$

$$= \sum_{n=0}^{\infty} (-1)^{n+1} \underbrace{\frac{1}{2n+1}}_{a_n}$$

Alt. Series

$\sum (-1)^{n+1} a_n$

<u>Alt. series test</u>: $a_n \geq 0$

$a_n \to 0$ as $n \to \infty$

a_n decreasing

<u>Our case</u>

$\frac{1}{2n+1} \geq 0$ ✓

$\frac{1}{2n+1} \to 0$ as $n \to \infty$ ✓

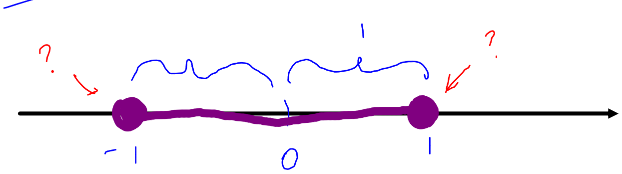
$\frac{1}{2n+1}$ is decreasing ✓

$\therefore$ AST $\Rightarrow$ the series converges.

<u>$x = 1$</u>: Subst. into P.S.

$$\sum_{n=0}^{\infty} (-1)^n \frac{1}{2n+1}$$

<u>Similar</u>. Converges by AST.



$\therefore$ our P.S. for $\arctan(x)$ converges on $[-1, 1]$.

$$\frac{1}{1+x} = \frac{1}{1-(-x)} = \sum_{n=0}^{\infty} (-x)^n, \quad |x|<1$$

$$= \sum_{n=0}^{\infty} (-1)^n x^n.$$ radius of conv. = 1

<u>ln(1+x):</u>

$$\ln(1+x) + C = \int \frac{1}{1+x}\,dx$$

$$= \int \sum_{n=0}^{\infty} (-1)^n x^n \, dx$$

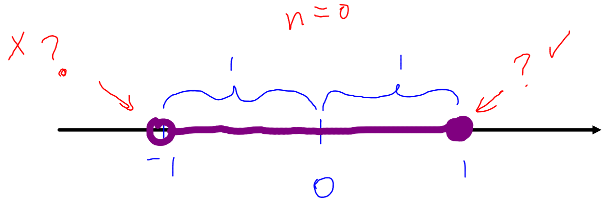
$$= \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1}$$

radius of conv. is 1.

<u>C = ?</u> Subst x = 0.

$$0 + C = 0$$

$$\therefore \ln(1+x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1}$$



<u>You do it:</u> <u>x = -1</u> diverges

<u>x = 1</u> converges.

$$\therefore \ln(1+x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1} \quad \text{for}$$

$$-1 < x \le 1.$$

$\therefore \quad \ln(1+u) = \sum_{n=0}^{\infty} (-1)^n \frac{u^{n+1}}{n+1}$ for $-1 < u \leq 1.$

$$\ln(1+\underbrace{x^3}_{u}) : = \sum_{n=0}^{\infty} (-1)^n \frac{(x^3)^{n+1}}{n+1},$$

for $-1 < x^3 \leq 1$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{x^{3n+3}}{n+1},$$

for $-1 < x \leq 1.$

$$\underbrace{\ln(1+x^3)} = \underbrace{\sum_{n=0}^{\infty} (-1)^n \frac{x^{3n+3}}{n+1}},$$

for $-1 < x \le 1$.

$$x^2 \underbrace{\ln(1+x^3)} = x^2 \cdot \sum_{n=0}^{\infty} (-1)^n \frac{x^{3n+3}}{n+1}$$

for $-1 < x \le 1$.

$$= \sum_{n=0}^{\infty} (-1)^n \frac{x^{3n+5}}{n+1}$$

for $-1 < x \le 1$.