14 More Test 4 Practice Problems Solutions	15 EMCF35 due at 9am Notes: page, 4-per, video notes, video Homework 12 due in lab/workshop	16	17 EMCF36 due at 9am Notes: page, 4-per Homework 13 posted	18 Video: Taylor Error Estimate Example	19 EMCF37 due at 9am Notes: page, 4-per, video notes, video Quiz in lab/workshop	20 Quiz 12 closes (11.1-11.4) 2012 Online Test 4 Video Review Slides
21 Note: Homework 13 is not due until the 29th!! Please consider taking the time to complete this survey.	22 EMCF38 due at 9am Blank Slides: page, 4-per, video notes, video	23 Complete the Online Teacher Evaluation by May 3 to Receive 5 Bonus Points	24 EMCF39 due at 9am Notes: page, 4-per	25 Final Exam Review Video (3 hours) Notes	26 EMCF40 due at 9am Blank slides: page, 4-per (in-class partial review for test 4) Quiz in lab/workshop	27 Quiz 13 closes (11.5-11.6) Test 4 starts
28	(*) 29 EMCF41 due at 9am Homework 13 due in lab/workshop Last day of class	30 Practice Test 4 Closes	May 1	2	3	4 Quiz 14 closes (11.7-11.8)
5	6 Final Exam Starts	7	8	9	10	11

Practice Final Closes.

Partial Test 4 Review

Infinite Series:
- Convergence, divergence, absolute convergence, conditional convergence.
- Alternating series and alternating series test.
- Convergence tests for series with nonnegative terms - integral test, comparison test, limit comparison test, ratio test, root test.
- Special series (p-series, geometric series).

Primarily Series

L'Hospital's Rule:
- Indeterminant forms.
- Applying the theorem.

Improper Integrals:
- Identification.
- Computation using proper notation.

Taylor Polynomial Approximation:
- Formula for Taylor polynomials.
- Taylor polynomials for simple functions.
- Error estimation and prediction of n to satisfy an error bound.

12 questions.
8 QUICK multiple choice (40 points).
4 written (60 points).

series tests
L'Hospital's
Improper Int.
Taylor Poly & Remainder

Example: Give the value of

$$\sum_{n=2}^{\infty}\frac{3-2^n}{5^n} = \sum_{n=2}^{\infty}\left(\frac{3}{5^n}-\left(\frac{2}{5}\right)^n\right)$$

$$= 3\sum_{n=2}^{\infty}\left(\frac{1}{5}\right)^n - \sum_{n=2}^{\infty}\left(\frac{2}{5}\right)^n$$

$$= 3\,\frac{(1/5)^2}{1-1/5} - \frac{(2/5)^2}{1-2/5}$$

$$= \dots$$

Geom Series

$$\sum_{n=k}^{\infty} r^n = \frac{r^k}{1-r}, \quad |r|<1$$

$$\sum_{n=2}^{\infty}\frac{\cos(n\pi)}{4^n}$$

Popper 32

1. Give this value.

Example: Determine whether the series converges or diverges.

Converges. ratio $\sum_{n=1}^{\infty}\frac{9^n}{n!}$

$$\sum_{n=1}^{\infty}\frac{1}{\sqrt[4]{n^3}} \qquad \sum_{n=1}^{\infty}\frac{2^n}{n^3} \qquad \sum_{n=1}^{\infty}\left(\frac{1}{n+1}-\frac{1}{n}\right) \qquad \sum_{n=1}^{\infty}\frac{3^{2n}}{n!}$$

Divergent p series $p = 3/4$

Diverge $2^n >> n^3$ terms $\not\to 0$

Popper 32

2. 0=converge, 1=diverge

$$\sum_{n=1}^{\infty}\cos(\pi n) \qquad \sum_{n=1}^{\infty}\frac{\sqrt{n}}{n} \qquad \sum_{n=1}^{\infty}\frac{(-1)^{n-1}n^2}{3n^3+1}$$

Diverges terms $\not\to 0$ $\cos(n\pi)=(-1)^n$

Diverges. $\sum\frac{1}{n^{1/2}}$ div. p series

Popper 32

3. 0=converge, 1=diverge

$$\sum_{n=0}^{\infty}3\left(-\frac{1}{2}\right)^n \qquad \sum_{n=2}^{\infty}\frac{1}{n(\ln n)^2} \qquad \sum_{n=2}^{\infty}\frac{3}{(n+1)\ln(n+2)}$$

Converges $3\sum_{n=0}^{\infty}\left(-\frac{1}{2}\right)^n$ Geom Series $r=-\frac{1}{2}$ $|r|<1$.

Converges. Integral test $\int_2^{\infty}\frac{1}{x(\ln(x))^2}dx < \infty$

Diverges. Like $\sum_{n=2}^{\infty}\frac{1}{n\ln(n)}$ Int test $\int_2^{\infty}\frac{1}{x\ln(x)}dx = \infty$

Example: Determine whether the series converges or diverges.

$\sum_{n=1}^{\infty} ne^{-n^3}$ Converge

$\sum_{n=1}^{\infty}\left(\frac{n}{n+1}\right)^n$ diverge terms $\not\to 0$

$\sum_{n=1}^{\infty}\frac{1}{n^3+1}$ Conv. p series

$\sum_{n=0}^{\infty}\left(\frac{2}{9}\right)^n$ Converge. geom series

$\sum_{n=1}^{\infty}\frac{n^2}{2^n}$ Converges. ratio or root

$\sum_{n=2}^{\infty}\frac{10n^2+n-2}{2n^6+7n-1}$ Converge. LCT with $\sum \frac{1}{n^4}$

$\sum_{n=1}^{\infty}\frac{n^2+3n-2}{\sqrt{4n^9+n-1}}$ Converge LCT with $\sum \frac{1}{n^{5/2}}$

Example: Determine whether the series converges absolutely, converges conditionally, or diverges. Show your work.

$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}\sqrt{n}}{n+3}$ ABS? NO COND? Yes AST

$\sum_{n=1}^{\infty}\frac{\cos \pi n}{n^2}$ ABS? Yes

$\sum_{n=0}^{\infty}\left(4(-1)^n\left(\frac{n}{n+3}\right)^n\right)$ Diverges Terms $\not\to 0$.

$\sum_{n=2}^{\infty}\frac{(-1)^n n!}{(n+1)!}$ ABS? NO COND? Yes AST

$\sum_{n=0}^{\infty}\frac{3(-1)^n}{\sqrt{3n^2+2n+1}}$ ABS? NO COND? Yes AST

$\sum_{n=2}^{\infty}\frac{(-1)^n 3^n}{n!}$ ABS? Yes Ratio Test

$\sum_{n=0}^{\infty}\frac{(-1)^n 10n^2}{3^n}$ ABS? Yes root or ratio

$\sum_{n=0}^{\infty}\frac{3n(-1)^n}{\sqrt{3n^2+2n+1}}$ Terms $\not\to 0$

$\sum_{n=2}^{\infty}\frac{\cos(\pi n)n^n}{n!}$ Diverges $n^n >> n!$ Terms $\not\to 0$.

Popper 32

4. 0=converge, 1=diverge

Example: Determine whether the limit is in indeterminant form. Then compute the limit.

$\frac{\infty}{\infty}$ L'H ✓ $\lim_{n\to\infty}\frac{\ln(n+4)}{n+2} = 0$

$\frac{0}{0}$ L'H $\lim_{x\to 0}\frac{x-\sin(2x)}{x+\sin(2x)} = -\frac{1}{3}$

NO L'H X $\lim_{x\to\infty}\frac{\sin(2x)}{3x} = 0$

$\frac{0}{0}$ L'H ✓ $\lim_{x\to 0}\frac{1+x-e^x}{x(e^x-1)}$

$\frac{0}{0}$ L'H ✓ $\lim_{x\to 0}\frac{e^{x^2}-1}{2x^2}$

$\frac{0}{0}$ L'H $\lim_{x\to 0}\frac{\arctan(4x)}{x} = 4$

1^∞ L'H X $\lim_{n\to\infty}\left(1+\frac{3}{n}\right)^{2n}$ "e^6"

$\frac{0}{0}$ L'H ✓ Twice $\lim_{x\to 0}\frac{3e^{x/3}-(3+x)}{x^2} = \frac{\frac{1}{3}}{2} = \frac{1}{6}$

$\lim_{n\to\infty}\left[\left(1+\frac{a}{n}\right)^n\right]^2 = \left(e^a\right)^2 = e^{2a}$

Example: Determine whether the integral is improper. If it is improper, then evaluate it using proper limit notation.

$\int_0^{27} x^{-2/3}\,dx$ Improper b/c of 0

$\int_{-2}^{0}\frac{1}{x+1}\,dx$ Improper b/c $x=-1$ is in $[-2,0]$. Split

$\int_1^4 \frac{1}{x+1}\,dx$ Not improper

$\int_0^4 \frac{1}{\sqrt{4-x}}\,dx$ Improper b/c of 4

$\int_1^{\infty}\frac{1}{x^2+1}\,dx$ Improper b/c of ∞

$\lim_{t\to 4^-}\int_0^t \frac{1}{\sqrt{4-x}}\,dx = \ldots$

Example: Give the 4th degree Taylor polynomial centered at 0 for $\sin(2x)$.

$f(x) = \sin(2x)$ $\quad f(0) = 0$

$f'(x) = 2\cos(2x)$ $\quad f'(0) = 2$

$f''(x) = -4\sin(2x)$ $\quad f''(0) = 0$

$f'''(x) = -8\cos(2x)$ $\quad f'''(0) = -8$

$f^{(4)}(x) = 16\sin(2x)$ $\quad f^{(4)}(0) = 0$

$P_4(x) = 2x - \frac{8}{6}x^3$

Example: Give the 4th degree Taylor polynomial centered at 0 for $\cos(3x)$.

You

Example: Give the 4th degree Taylor polynomial centered at 0 for $\exp(-2x)$.

you

Example: $f(-1) = -2$, $f'(-1) = 3$, $f''(-1) = 4$ and $f'''(-1) = 3/2$. Give the 3rd degree Taylor polynomial centered at -1.

$$f(-1) + f'(-1)(x+1) + \frac{f''(-1)}{2}(x+1)^2 + \frac{f'''(-1)}{6}(x+1)^3$$

$$= -2 + 3(x+1) + 2(x+1)^2 + \frac{1}{4}(x+1)^3$$

Example: Give the smallest value of n so that the n^{th} degree Taylor polynomial centered at 0 approximates exp(-2) within 10^{-2}.

unknown.

$f(x) = e^x$

error is given

We want

$$|f(-2) - P_{n,0}(-2)| \le \frac{1}{10}$$

we know

$$|f(-2) - P_{n,0}(-2)| \le \frac{M^{=1}}{(n+1)!}|-2-0|^{n+1} \le \frac{1}{10}$$

Force

where

$$M \ge \underset{-2 \le x \le 0}{|f^{(n+1)}(x)|} = |e^x| = e^x$$

<u>Note</u>: $f(x) = e^x \Rightarrow f^{(n+1)}(x) = e^x$

$\therefore$ $M=1$ <u>works</u>

largest on $[-2,0]$ at $x=0$.

Let's find n so that $\dfrac{2^{n+1}}{(n+1)!} \le \dfrac{1}{10}$

	n	$\frac{2^{n+1}}{(n+1)!}$
<u>No</u>	4	$\frac{32}{120}$
<u>Yes</u>	5	$\frac{64}{720} < \frac{1}{10}$
	6	

$2^5 = 32$ $\quad 2^6 = 64$

$5! = 120$ $\quad 6! = 720$

$n = 5$