14 More Test 4 Practice Problems Solutions	15 EMCF35 due at 9am Notes: page, 4-per, video notes, video Homework 12 due in lab/workshop	16	17 EMCF36 due at 9am Notes: page, 4-per Homework 13 posted	18 Video: Taylor Error Estimate Example	19 EMCF37 due at 9am Notes: page, 4-per, video notes, video Quiz in lab/workshop	20 Quiz 12 closes (11.1-11.4) 2012 Online Test 4 Video Review Slides
21 **Note: Homework 13 is not due until the 29th!!** Please consider taking the time to complete this survey.	22 EMCF38 due at 9am Blank Slides: page, 4-per, video notes, video	**23** Complete the Online Teacher Evaluation by May 3 to Receive 5 Bonus Points	24 EMCF39 due at 9am Notes: page, 4-per	25 Final Exam Review Video (3 hours) Notes	26 EMCF40 due at 9am Blank slides: page, 4-per (in-class partial review for test 4) Quiz in lab/workshop	27 Quiz 13 closes (11.5-11.6) **Test 4 starts**
28	(*) 29 EMCF41 due at 9am Homework 13 due in lab/workshop **Last day of class**	30 Practice Test 4 Closes	May 1	2	3	4 Quiz 14 closes (11.7-11.8)
5	6 **Final Exam Starts**	7	8	9	10	11

Practice Final Closes.

Partial Test 4 Review

Infinite Series:

- Convergence, divergence, absolute convergence, conditional convergence.
- Alternating series and alternating series test.
- Convergence tests for series with nonnegative terms - integral test, comparison test, limit comparison test, ratio test, root test.
- Special series (p-series, geometric series).

L'Hospital's Rule:

- Indeterminant forms.
- Applying the theorem.

Improper Integrals:

- Identification.
- Computation using proper notation.

Taylor Polynomial Approximation:

- Formula for Taylor polynomials.
- Taylor polynomials for simple functions.
- Error estimation and prediction of n to satisfy an error bound.

12 questions.
8 QUICK multiple choice (40 points).
4 written (60 points).

primarily series

series tests
L'Hospital's
Improper Int.
Taylor Poly + Remainder

Example: Give the value of

$$\sum_{n=2}^{\infty} \frac{3-2^n}{5^n} = \sum_{n=2}^{\infty}\left(\frac{3}{5^n} - \left(\frac{2}{5}\right)^n\right)$$

$$= 3\sum_{n=2}^{\infty}\left(\frac{1}{5}\right)^n - \sum_{n=2}^{\infty}\left(\frac{2}{5}\right)^n$$

$$= 3\,\frac{(1/5)^2}{1-1/5} - \frac{(2/5)^2}{1-2/5}$$

$$= \ldots$$

Geom series

$$\sum_{n=k}^{\infty} r^n = \frac{r^k}{1-r}, \quad |r|<1$$

$$\sum_{n=2}^{\infty} \frac{\cos(n\pi)}{4^n}$$

Popper 32

1. Give this value.

Example: Determine whether the series converges or diverges.

Converges. ratio
$$\sum_{n=1}^{\infty} \frac{9^n}{n!}$$

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt[4]{n^3}} \qquad \sum_{n=1}^{\infty} \frac{2^n}{n^3} \qquad \sum_{n=1}^{\infty} \left(\frac{1}{n+1} - \frac{1}{n} \right) \qquad \sum_{n=1}^{\infty} \frac{3^{2n}}{n!}$$

Divergent p series $p = 3/4$

Diverge $2^n >> n^3$ terms $\not\to 0$

Popper 32

2. 0=converge, 1=diverge

$$\sum_{n=1}^{\infty} \cos(\pi n) \qquad \sum_{n=1}^{\infty} \frac{\sqrt{n}}{n} \qquad \sum_{n=1}^{\infty} \frac{(-1)^{n-1} n^2}{3n^3 + 1}$$

Diverges terms $\not\to 0$ $\cos(n\pi) = (-1)^n$

Diverges. $\sum \frac{1}{n^{1/2}}$ div. p series

Popper 32

3. 0=converge, 1=diverge

$$\sum_{n=0}^{\infty} 3\left(-\frac{1}{2}\right)^n \qquad \sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2} \qquad \sum_{n=2}^{\infty} \frac{3}{(n+1)\ln(n+2)}$$

Converges $3 \sum_{n=0}^{\infty} \left(-\frac{1}{2}\right)^n$ Geom Series $r = -\frac{1}{2}$ $|r| < 1$.

Converges. Integral test $\int_2^{\infty} \frac{1}{x(\ln(x))^2}\,dx < \infty$

Diverges, like $\sum_{n=2}^{\infty} \frac{1}{n \ln(n)}$ Int test $\int_2^{\infty} \frac{1}{x \ln(x)}\,dx = \infty$

Example: Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} n e^{-n^3}$$

converge

$$\sum_{n=1}^{\infty} \left(\frac{n}{n+1} \right)^n$$

diverge

terms $\not\to 0$

$$\sum_{n=1}^{\infty} \frac{1}{n^3+1}$$

conv. p series

$$\sum_{n=0}^{\infty} \left(\frac{2}{9} \right)^n$$

converge, geom series

$$\sum_{n=1}^{\infty} \frac{n^2}{2^n}$$

converges. ratio or root

$$\sum_{n=2}^{\infty} \frac{10n^2+n-2}{2n^6+7n-1}$$

converge. LCT with $\sum \frac{1}{n^4}$

$$\sum_{n=1}^{\infty} \frac{n^2+3n-2}{\sqrt{4n^9+n-1}}$$

converge LCT with $\sum \frac{1}{n^{5/2}}$

Example: Determine whether the series converges absolutely, converges conditionally, or diverges. Show your work.

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}\sqrt{n}}{n+3}$$

ABS? NO
COND? Yes AST

$$\sum_{n=1}^{\infty}\frac{\cos \pi n}{n^2}$$

ABS? Yes

$$\sum_{n=0}^{\infty}\left(4(-1)^n\left(\frac{n}{n+3}\right)^n\right)$$

Diverges Terms $\not\to 0$.

$$\sum_{n=2}^{\infty}\frac{(-1)^n n!}{(n+1)!}$$

ABS? NO
COND? Yes AST

$$\sum_{n=0}^{\infty}\frac{3(-1)^n}{\sqrt{3n^2+2n+1}}$$

ABS? NO
COND? Yes AST

$$\sum_{n=2}^{\infty}\frac{(-1)^n 3^n}{n!}$$

ABS? Yes
Ratio Test

$$\sum_{n=0}^{\infty}\frac{(-1)^n 10n^2}{3^n}$$

ABS? Yes
root or ratio

$$\sum_{n=0}^{\infty}\frac{3n(-1)^n}{\sqrt{3n^2+2n+1}}$$

Terms $\not\to 0$

$$\sum_{n=2}^{\infty}\frac{\cos(\pi n)n^n}{n!}$$

Diverges
$n^n >> n!$,
Terms $\not\to 0$.

Popper 32

4. 0=converge, 1=diverge

Example: Determine whether the limit is in indeterminant form. Then compute the limit.

$\frac{\infty}{\infty}$ L'H ✓

$$\lim_{n\to\infty} \frac{\ln(n+4)}{n+2} = 0$$

$\frac{0}{0}$ L'H

$$\lim_{x\to 0} \frac{x-\sin(2x)}{x+\sin(2x)} = \frac{-1}{3}$$

NO L'H X

$$\lim_{x\to\infty} \frac{\sin(2x)}{3x} = 0$$

$\frac{0}{0}$ L'H ✓

$$\lim_{x->0} \frac{1+x-e^x}{x(e^x-1)}$$

$\frac{0}{0}$ L'H ✓

$$\lim_{x->0} \frac{e^{x^2}-1}{2x^2}$$

$\frac{0}{0}$ L'H

$$\lim_{x->0} \frac{\arctan(4x)}{x} = 4$$

1^∞ L'H X

$$\lim_{n\to\infty} \left(1+\frac{3}{n}\right)^{2n} = e^6$$

$\frac{0}{0}$ L'H ✓ Twice

$$\lim_{x->0} \frac{3e^{x/3}-(3+x)}{x^2} = \frac{\frac{1}{3}}{2} = \frac{1}{6}$$

$$\lim_{n\to\infty} \left[\left(1+\frac{a}{n}\right)^n\right]^2 = \left(e^a\right)^2 = e^{2a}$$

Example: Determine whether the integral is improper. If it is improper, then evaluate it using proper limit notation.

$$\int_0^{27} x^{-2/3}\,dx$$

Improper b/c of

$$\int_{-2}^{0} \frac{1}{x+1}\,dx$$

Improper b/c $x=-1$ is in $[-2, 0]$.

split

$$\int_{1}^{4} \frac{1}{x+1}\,dx$$

Not improper

$$\int_0^4 \frac{1}{\sqrt{4-x}}\,dx$$

Improper b/c of

$$\lim_{t\to 4^-} \int_0^t \frac{1}{\sqrt{4-x}}\,dx = \ldots$$

$$\int_1^{\infty} \frac{1}{x^2+1}\,dx$$

Improper b/c of

Example: Give the 4th degree Taylor polynomial centered at 0 for sin(2*x*).

$f(x) = \sin(2x)$ $\qquad$ $f(0) = 0$

$f'(x) = 2\cos(2x)$ $\qquad$ $f'(0) = 2$

$f''(x) = -4\sin(2x)$ $\qquad$ $f''(0) = 0$

$f'''(x) = -8\cos(2x)$ $\qquad$ $f'''(0) = -8$

$f^{(4)}(x) = 16\sin(2x)$ $\qquad$ $f^{(4)}(0) = 0$

$P_4(x) = 2x - \frac{8}{6}x^3$

Example: Give the 4th degree Taylor polynomial centered at 0 for $\cos(3x)$.

You

Example: Give the 4th degree Taylor polynomial centered at 0 for $\exp(-2x)$.

you

Example: $f(-1) = -2$, $f'(-1) = 3$, $f''(-1) = 4$ and $f'''(-1) = 3/2$. Give the 3rd degree Taylor polynomial centered at -1.

$$f(-1) + f'(-1)(x+1) + \frac{f''(-1)}{2}(x+1)^2 + \frac{f'''(-1)}{6}(x+1)^3$$

$$= -2 + 3(x+1) + 2(x+1)^2 + \frac{1}{4}(x+1)^3$$

Example: Give the smallest value of n so that the n^{th} degree Taylor polynomial centered at 0 approximates exp(-2) within 10^{-2}.

$f(x) = e^x$

unknown.

error is given

We want

$$|f(-2) - P_{n,0}(-2)| \le \frac{1}{10}$$

We know

$$|f(-2) - P_{n,0}(-2)| \le \frac{M^{=1}}{(n+1)!}|-2-0|^{n+1} \le \frac{1}{10}$$

Force

where

$$M \ge |f^{(n+1)}(x)| = |e^x| = e^x$$

$-2 \le x \le 0$

Note: $f(x) = e^x \Rightarrow f^{(n+1)}(x) = e^x$

$\therefore$ M=1 works

largest on $[-2,0]$ at $x=0$.

Let's find n so that $\frac{2^{n+1}}{(n+1)!} \le \frac{1}{10}$

	n	$\frac{2^{n+1}}{(n+1)!}$
No	4	$\frac{32}{120}$
Yes	5	$\frac{64}{720} < \frac{1}{10}$
	6	

$2^5 = 32$ $\quad 2^6 = 64$

$5! = 120$ $\quad 6! = 720$

$n = 5$