Selected solutions to problems in 7.7...

3 (a) $\cos^{-1}\left(-\frac{1}{2}\right)$;

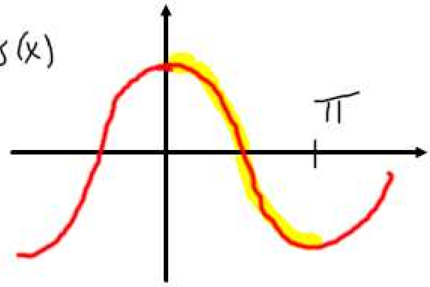
value btwn 0 and π where cosine is $-\frac{1}{2}$

Solve $\cos(x) = -\frac{1}{2}$

$x = \frac{2\pi}{3}$.

$\therefore \cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$

$y = \cos(x)$



restricted to $[0, \pi]$.

$\arccos(x)$ defined on $[-1, 1]$.

range: $[0, \pi]$.

[illegible] (a) $\sin\left(2\cos^{-1}\left[\frac{1}{2}\right]\right)$;

First, $\cos^{-1}\left(\frac{1}{2}\right) = x$

where $0 \leq x \leq \pi$

and $\cos(x) = \frac{1}{2}$.

$\boxed{x = \frac{\pi}{3}} = \cos^{-1}\left(\frac{1}{2}\right)$

$\sin\left(2 \cdot \frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$.

43. $f(x) = \sec^{-1}(2x^2)$.

$$f'(x) = \frac{1}{|2x^2|\sqrt{(2x^2)^2 - 1}} \cdot 4x$$

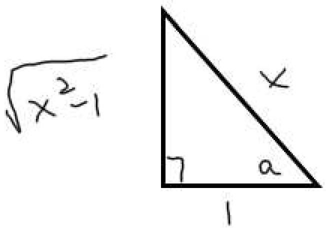
$$= \frac{2}{x\sqrt{4x^4 - 1}}.$$

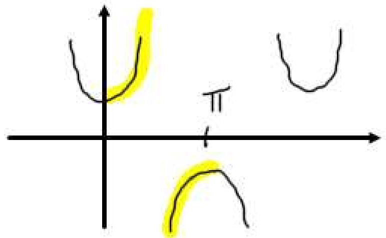
Q: $\frac{d}{dx}\sec^{-1}(x)$?

in the book

$$\frac{d}{dx}\sec^{-1}(x) = \frac{1}{\sec(a)\tan(a)} = \frac{1}{|x|\sqrt{x^2 - 1}}$$

where $\sec(a) = x$





26. $\theta = \sin^{-1}\left(\frac{r}{r+1}\right)$.

$$\frac{d}{dx}\arcsin(u) = \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}$$

$$\frac{d\theta}{dr} = \frac{1}{\sqrt{1-\left(\frac{r}{r+1}\right)^2}} \cdot \frac{d}{dr}\underbrace{\left(\frac{r}{r+1}\right)}_{\text{quotient rule.}}.$$

do it.

31. $f(x)=\sqrt{c^2-x^2}+c\sin^{-1}\left(\frac{x}{c}\right), c>0.$

C is a positive constant.

$$f'(x)=\frac{1}{2}\left(c^2-x^2\right)^{-\frac{1}{2}}\cdot(-2x)+c\cdot\frac{1}{\sqrt{1-\left(x/c\right)^2}}\cdot\frac{1}{c}$$

$$\boxed{\begin{aligned}&\underline{\text{Note}}:\ \frac{x}{c}=\frac{1}{c}\cdot x\\ &\Rightarrow \frac{d}{dx}\left(\frac{x}{c}\right)=\frac{1}{c}.\end{aligned}}$$

41. $\int_0^{1/\sqrt{2}}\frac{dx}{\sqrt{1-x^2}}.$ $=\arcsin(x)\Big|_0^{\sqrt{2}/2}$

$$=\arcsin\left(\frac{\sqrt{2}}{2}\right)-\arcsin(0)$$

$$=\frac{\pi}{4}-0=\frac{\pi}{4}.$$

46. $\int_2^5 \frac{dx}{9+(x-2)^2}$.

$$\int \frac{1}{a^2+u^2}\,du = \frac{1}{a}\arctan\left(\frac{u}{a}\right) + C$$

$$= \frac{1}{3}\arctan\left(\frac{x-2}{3}\right)\Big|_2^5$$

$$= \frac{1}{3}\left[\arctan(1) - \arctan(0)\right]$$

$$= \frac{1}{3}\left[\frac{\pi}{4} - 0\right]$$

$$= \pi/12.$$

$$\frac{d}{dx}\arctan\left(\frac{x}{a}\right) = \frac{1}{1+\left(\frac{x}{a}\right)^2}\cdot\frac{1}{a}$$

$$= \frac{a^2}{a^2+x^2}\cdot\frac{1}{a}$$

$$\therefore \quad \frac{d}{dx}\,\frac{1}{a}\arctan\left(\frac{x}{a}\right) = \frac{1}{a^2+x^2}$$

60. $\int \frac{\tan^{-1}x}{1+x^2}\,dx$. $= \frac{1}{2}\left(\arctan(x)\right)^2 + C$