Selected problems from 7.8...

5. $y = \dfrac{\sinh x}{\cosh x - 1}$.

$$y' = \frac{(\cosh(x) - 1)\cdot\cosh(x) - \sinh(x)\sinh(x)}{(\cosh(x) - 1)^2}$$

$$= \frac{\cosh^2(x) - \cosh(x) - \sinh^2(x)}{(\cosh(x) - 1)^2}$$

$$= \frac{1 - \cosh(x)}{(\cosh(x) - 1)^2} = \frac{1}{1 - \cosh(x)}$$

17. $y = (\sinh x)^x$. $= (\sinh(x))^x$

$y' = ??$.

$\ln(y) = \ln\left((\sinh(x))^x\right)$

$\ln(y) = x \ln(\sinh(x))$

$\frac{1}{y} y' = \ln(\sinh(x)) + x \frac{\cosh(x)}{\sinh(x)}$

$y' = (\sinh(x))^x \left[\ln(\sinh(x)) + x \coth(x)\right]$.

Find the absolute extreme values.

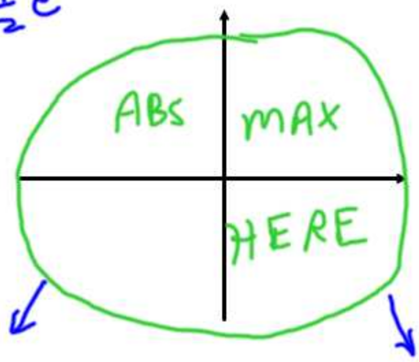
27. $y = -5\cosh x + 4\sinh x$. $= -5\left(\dfrac{e^{x}+e^{-x}}{2}\right) + 4\left(\dfrac{e^{x}-e^{-x}}{2}\right)$

$$= -\tfrac{1}{2}e^{x} - \tfrac{9}{2}e^{-x}$$

Note

$$\lim_{x\to-\infty}\Big(\underbrace{-\tfrac{1}{2}e^{x}}_{\to 0} - \underbrace{\tfrac{9}{2}e^{-x}}_{\to -\infty}\Big) = -\infty$$

$$\lim_{x\to\infty}\Big(\underbrace{-\tfrac{1}{2}e^{x}}_{\to -\infty} - \underbrace{\tfrac{9}{2}e^{-x}}_{\to 0}\Big) = -\infty$$



No abs. min.

Look for abs. max

$$y' = -\tfrac{1}{2}e^{x} + \tfrac{9}{2}e^{-x}$$

Set $y' = 0$ $\quad -\tfrac{1}{2}e^{x} + \tfrac{9}{2}e^{-x} = 0$

$$e^{x} = 9e^{-x} \longrightarrow e^{2x} = 9$$

$$2x = \ln(9)$$

$$x = \tfrac{1}{2}\ln(9) = \ln(3)$$

Abs. max. at $x = \ln(3)$.

$$y\Big|_{x=\ln(3)} = \left(-\tfrac{1}{2}e^{x} - \tfrac{9}{2}e^{-x}\right)\Big|_{x=\ln(3)}$$

$$= -\tfrac{3}{2} - \tfrac{9}{2}e^{-\ln(3)}$$

$$= -\tfrac{3}{2} - \tfrac{9}{2}\cdot\tfrac{1}{3} = -3$$

30. Verify that $y = A \cosh cx + B \sinh cx$ satisfies the equation $y'' - c^2 y = 0$.

$$y = A\cosh(cx) + B\sinh(cx)$$

$$y' = Ac\sinh(cx) + Bc\cosh(cx)$$

$$y'' = Ac^2\cosh(cx) + Bc^2\sinh(cx)$$

$$= c^2 y$$

$$\therefore \quad y'' - c^2 y = 0.$$

+ 39. $\int \frac{\sinh ax}{\cosh^2 ax}\,dx.$ $= \frac{1}{a}\int (\cosh(ax))^{-2} a\sinh(ax)\,dx$

$= \frac{1}{a}\int u^{-2}\,du$

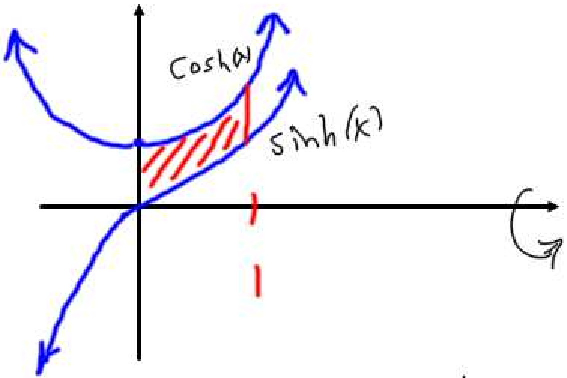
$u = \cosh(ax)$

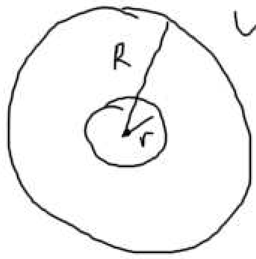
$du = a\sinh(ax)dx$

$= \frac{-1}{au} + C$

$= \frac{-1}{a\cosh(ax} + C$

49. Find the volume of the solid generated by revolving the region bounded by $y = \cosh x$ and $y = \sinh x$ between $x = 0$ and $x = 1$ about the x-axis.





Washer

$r = \sinh(x)$

$R = \cosh(x)$

$$\text{Volume} = \int_0^1 (\pi R^2 - \pi r^2)\,dx$$

$$= \int_0^1 (\pi \cosh^2(x) - \pi \sinh^2(x))\,dx$$

$$= \int_0^1 \pi\,dx = \pi x \Big|_0^1 = \pi.$$