Test 3 Online Review - Spring 2013

Sections 8.4 - 10.3

10 questions: 3 multiple choice and 7 written

- Trigonometric Substitution
- Partitial Fraction Decomposition
- Numerical Integration
- Polar Coordinates
- Parametric Curves
- Sets, Sequences, LUB, GLB, Monotonicity and Limits

Part I - Trigonometric Substitution

$a > 0$

For integrals involving...	Use the substitution...
$\sqrt{a^2 - x^2}$	$x = a\sin(\theta)$
$\sqrt{a^2 + x^2}$	$x = a\tan(\theta)$
$\sqrt{x^2 - a^2}$	$x = a\sec(\theta)$

$$\int \frac{1}{\sqrt{x^2+4}}\,dx = \int \frac{1}{\sqrt{\underline{4}\tan^2(\theta)+\underline{4}}}\, 2\sec^2(\theta)\,d\theta$$

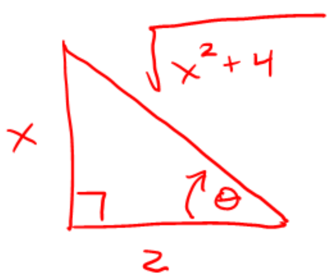
$\boxed{x = 2\tan(\theta)}$

$dx = 2\sec^2(\theta)\,d\theta$

$$= \int \frac{1}{2\sqrt{\underbrace{\tan^2(\theta)+1}_{=\sec^2(\theta)}}} \cdot 2\sec^2(\theta)\,d\theta$$

$\frac{x}{2} = \tan(\theta)$

$$= \int \frac{\sec^2(\theta)}{\sec(\theta)}\,d\theta = \int \sec(\theta)\,d\theta$$



$$= \ln\left|\sec(\theta) + \underline{\tan(\theta)}\right| + C$$

$\sec(\theta) = \frac{\sqrt{x^2+4}}{2}$

$$\boxed{= \ln\left|\tfrac{1}{2}\sqrt{x^2+4} + \tfrac{1}{2}x\right| + C}$$

$$= \ln\left|\sqrt{x^2+4} + x\right| + \tilde{C}$$

$$\ln\left|\frac{\sqrt{x^2+4}+x}{2}\right|$$

$$= \ln\left|\sqrt{x^2+4} + x\right| - \underline{\ln(2)}$$

absorbed.

$$\int \frac{2x^2}{\sqrt{9-x^2}}\,dx = \int \frac{18\sin^2(\theta)}{\sqrt{9-9\sin^2(\theta)}} \cdot 3\cos(\theta)\,d\theta$$

$$\boxed{x = 3\sin(\theta)}$$

$$dx = 3\cos(\theta)\,d\theta$$

$$\boxed{\frac{x}{3} = \sin(\theta)}$$

$$\boxed{\theta = \arcsin\left(\frac{x}{3}\right)}$$

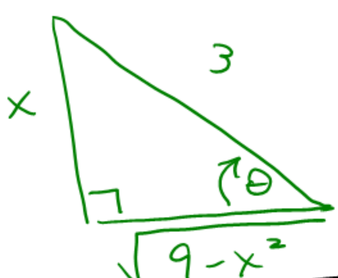
$$= \int \frac{18\sin^2(\theta)}{\cancel{3}\sqrt{\underbrace{1-\sin^2(\theta)}_{\cos^2(\theta)}}} \cdot \cancel{3}\cos(\theta)\,d\theta$$

$$= 18\int \sin^2(\theta)\,d\theta$$

Note:

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

$$= 18\int\left(\frac{1}{2} - \frac{1}{2}\cos(2\theta)\right)d\theta$$



$$\boxed{\cos(\theta) = \frac{1}{3}\sqrt{9-x^2}}$$

$$= 9\theta - \frac{9}{2}\sin(2\theta) + C$$

$$= 9\theta - 9\sin(\theta)\cos(\theta) + C$$

$$= 9\arcsin\left(\frac{x}{3}\right) - 9\cdot\frac{x}{3}\cdot\frac{\sqrt{9-x^2}}{3} + C$$

$$= 9\arcsin\left(\frac{x}{3}\right) - x\sqrt{9-x^2} + C.$$

Part II - Partial Fraction Decomposition

Setting: $\int \frac{p(x)}{q(x)}dx$, where $p(x)$ and $q(x)$ are polynomials

and the degree of $q(x)$ > degree of $p(x)$

Process:

1. Factor $q(x)$ into linear and irreducible quadratics

2. PFD : Rewrite $\frac{p(x)}{q(x)}$ in pieces from our factors.

3. Integrate the pieces to get $\int \frac{p(x)}{q(x)}dx$

$$\int \frac{3x-1}{x^2-x-6}\,dx =$$

Note:

$$\frac{3x-1}{x^2-x-6} = \boxed{\frac{3x-1}{(x-3)(x+2)}}$$

Do a PFD to $\frac{3x-1}{(x-3)(x+2)}$

Note: There is a very good chance that you will be asked to do a partial fraction decomposition.

$$\frac{3x-1}{(x-3)(x+2)} = \frac{A}{x-3} + \frac{B}{x+2}$$

$$3x-1 = A(x+2) + B(x-3)$$

$x=-2$: $-7 = -5B \Rightarrow B = \frac{7}{5}$

$x=3$: $8 = 5A \Rightarrow A = \frac{8}{5}$

$\therefore$ the PFD is $\boxed{\frac{3x-1}{(x-3)(x+2)} = \frac{8/5}{(x-3)} + \frac{7/5}{(x+2)}}$

Let's use this to integrate.

$$\int \frac{3x-1}{(x-3)(x+2)}\,dx = \int\left(\frac{8/5}{(x-3)} + \frac{7/5}{(x+2)}\right)dx$$

$$= \frac{8}{5}\ln|x-3| + \frac{7}{5}\ln|x+2| + C$$

$$\int \frac{3x-1}{\underbrace{(x-1)^2}_{\text{repeated linear}}\underbrace{\left(x^2+5\right)}_{\text{irreducible quadratic}}}\,dx =$$

PFD:

$$\frac{3x-1}{(x-1)^2(x^2+5)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{Cx+D}{x^2+5}$$

$$3x-1 = A(x-1)(x^2+5) + B(x^2+5) + (Cx+D)(x-1)^2$$

Killer x: $x=1$

$$2 = 6B \Rightarrow \boxed{B = \frac{1}{3}}$$

Plug in other x values.

$x=0$: $\boxed{-1 = -5A + 5B + D}$

$x=-1$: $\boxed{-4 = -12A + 6B - 4C + 4D}$

$x=2$: $\boxed{5 = 9A + 9B + 2C + D}$

4 equations with 4 unknowns.

We could also try the following:

$$3x-1 = A(x-1)(x^2+5) + B(x^2+5) + (Cx+D)(x-1)^2$$

we found $\boxed{B = \frac{1}{3}}$ using the killer x, $x=1$.

$$3x-1 = A(x^3 - x^2 + 5x - 5) + Bx^2 + 5B$$
$$+ (Cx+D)(x^2 - 2x + 1)$$

$$3x-1 = A(x^3 - x^2 + 5x - 5) + Bx^2 + 5B$$
$$+ Cx^3 + (D-2C)x^2 + (C-2D)x + D$$

$$3x-1 = (A+C)x^3 + (-A+B-2C+D)x^2 + (5A+C-2D)x$$
$$+ (-5A+5B+D)$$

$$0 = A + C$$
$$0 = -A + B - 2C + D$$
$$3 = 5A + C - 2D$$
$$-1 = -5A + 5B + D$$

Note: Whether we substitute individual x values or do this "grocery shopping", we end up with a system of equations.

Find A, B, C, D. . Then integrate.

$A = \frac{7}{18}$, $B = \frac{1}{3}$, $C = \frac{-7}{18}$, $D = \frac{-13}{18}$.

$$\int \frac{3x-1}{(x-1)^2(x^2+5)}\,dx = \int \left(\frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{Cx+D}{x^2+5} \right) dx$$

$$= A \ln|x-1| - \frac{B}{x-1} + \int \frac{Cx+D}{x^2+5}\,dx$$

$$= A \ln|x-1| - \frac{B}{x-1} + C \cdot \frac{1}{2} \int \frac{2x}{x^2+5}\,dx + D \int \frac{1}{x^2+5}\,dx$$

$$= A \ln|x-1| - \frac{B}{x-1} + \frac{C}{2} \ln|x^2+5| + \frac{D}{\sqrt{5}} \arctan\left(\frac{x}{\sqrt{5}}\right) + C$$

$$\boxed{a>0 \qquad \int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right) + C}$$

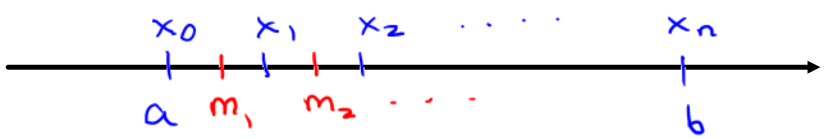
Part III - Numerical Integration

Tools for approximating $\int_a^b f(x)\,dx$

$$m_i = \frac{x_{i-1} + x_i}{2}$$

Set up: n is a natural number.

$$x_{i+1} - x_i = \frac{b-a}{n}$$



$$T_n = \frac{b-a}{2n}\left[f(x_0) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(x_n) \right]$$

$$M_n = \frac{b-a}{n}\left[f(m_1) + f(m_2) + \cdots + f(m_n) \right]$$

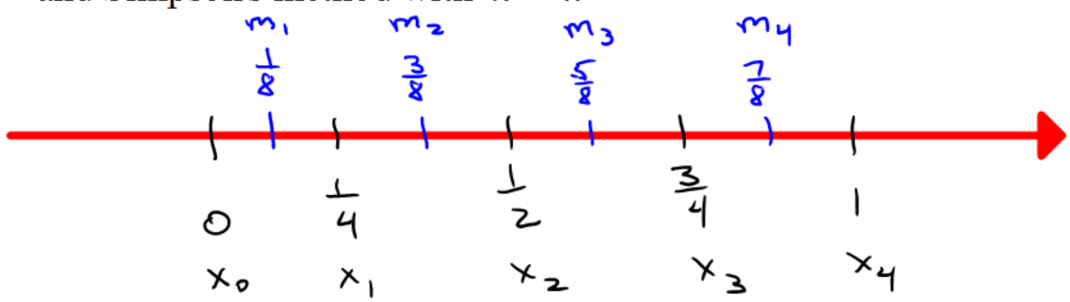
$$S_n = \frac{1}{3}T_n + \frac{2}{3}M_n$$

$$f(x) = \frac{x}{x+1} \qquad\qquad \frac{3/4}{\frac{3}{4}+1}$$

Estimate $\int_0^1 \frac{x}{x+1}\,dx$ using the trapezoid method, the midpoint method, and Simpson's method with $n = 4$.

$a = 0,\ b = 1$

$n = 4$



$$\frac{b-a}{n} = \frac{1}{4}$$

$$\frac{b-a}{2n} = \frac{1}{8}$$

$$T_4 = \frac{1}{8}\left(f(0) + 2f\left(\tfrac{1}{4}\right) + 2f\left(\tfrac{1}{2}\right) + 2f\left(\tfrac{3}{4}\right) + f(1)\right)$$

$$= \frac{1}{8}\left(0 + 2\cdot\frac{1}{5} + 2\cdot\frac{1}{3} + 2\cdot\frac{3}{7} + \frac{1}{2}\right)$$

$$= \frac{1}{8}\left(0 + \frac{2}{5} + \frac{2}{3} + \frac{6}{7} + \frac{1}{2}\right) =$$ harder than the exam. do the arithmetic.

$$M_4 = \frac{1}{4}\left(f\left(\tfrac{1}{8}\right) + f\left(\tfrac{3}{8}\right) + f\left(\tfrac{5}{8}\right) + f\left(\tfrac{7}{8}\right)\right)$$

Recall $f(x) = \frac{x}{x+1}$

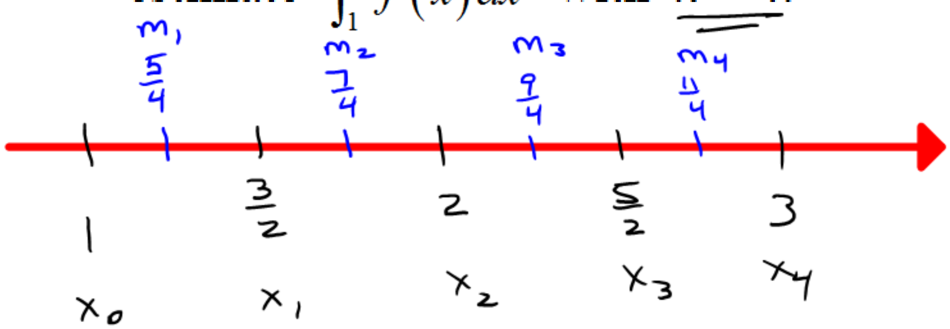
$$= \frac{1}{4}\left(\frac{1}{9} + \frac{3}{11} + \frac{5}{13} + \frac{7}{15}\right) =$$ ditto.

$$S_4 = \frac{1}{3}T_4 + \frac{2}{3}M_4.$$

Use the midpoint, trapezoid and Simpson's methods to

$a=1,\ b=3$ $\qquad \frac{b-a}{n} = \frac{1}{2}$ $\qquad \frac{b-a}{2n} = \frac{1}{4}$

estimate $\int_1^3 f(x)\,dx$ with $\underline{n=4}$.



x	$f(x)$
0	1
.25	.8
.5	.7
.75	.7
1	.6
1.25	.5
1.5	.4
1.75	.3
2	.3
2.25	.2
2.5	.1
2.75	0
3	0
3.25	.1
3.5	.2
3.75	.3
4	.4

$$M_4 = \frac{1}{2}\left(f\left(\frac{5}{4}\right) + f\left(\frac{7}{4}\right) + f\left(\frac{9}{4}\right) + f\left(\frac{11}{4}\right)\right)$$

$$= \frac{1}{2}\left(.5 + .3 + .2 + 0\right)$$

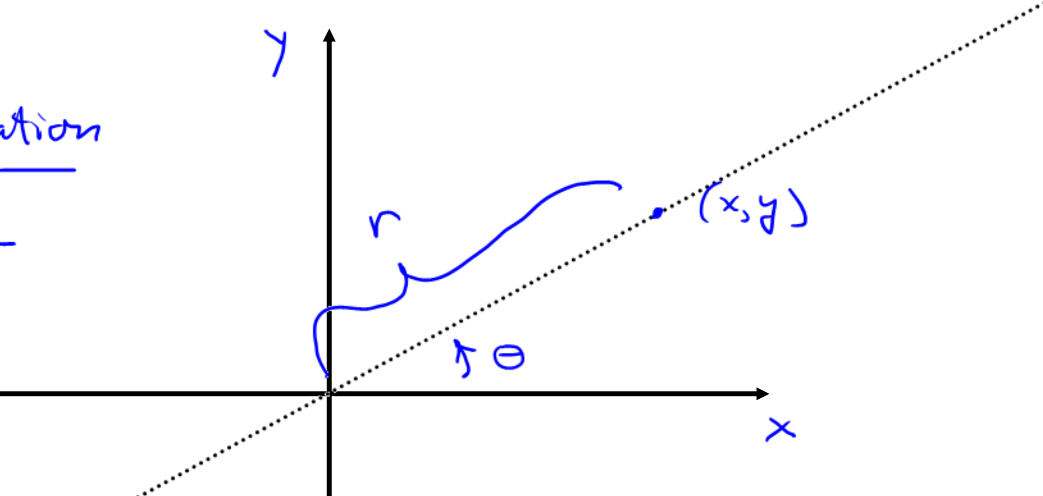
$$= \frac{1}{2} = .5$$

$$T_4 = \frac{1}{4}\left(f(1) + 2f\left(\frac{3}{2}\right) + 2f(2) + 2f\left(\frac{5}{2}\right) + f(3)\right)$$

$$= \frac{1}{4}\left(.6 + .8 + .6 + .2 + 0\right) = \frac{2.2}{4} = .55$$

$$S_4 = \frac{1}{3}T_4 + \frac{2}{3}M_4 = \frac{1}{3}\cdot(.55) + \frac{2}{3}(.5) = \underline{\underline{??}}$$

Part IV - Polar Coordinates



Standard Representation

$r = \sqrt{x^2 + y^2}$

$\theta = \arctan\left(\frac{y}{x}\right)$

provided (x, y) lies in Q1 or Q4.

Always $x = r\cos(\theta)$

$y = r\sin(\theta)$

Note: There are infinitely many polar representations for points in the xy plane. The standard representation is just one of these.

Note: polar coord. are written in the form $[r, \theta]$.

Give $\underbrace{\text{rectangular coordinates}}_{(x,y)}$ for the polar point $\left[-2, \frac{\pi}{3}\right]$.

$$\left[-2, \frac{\pi}{3}\right] = [r, \theta]$$

$$x = r\cos(\theta) = -2\cos\left(\frac{\pi}{3}\right) = -1$$

$$y = r\sin(\theta) = -2\sin\left(\frac{\pi}{3}\right) = -\sqrt{3}$$

$$\left(-1, -\sqrt{3}\right)$$

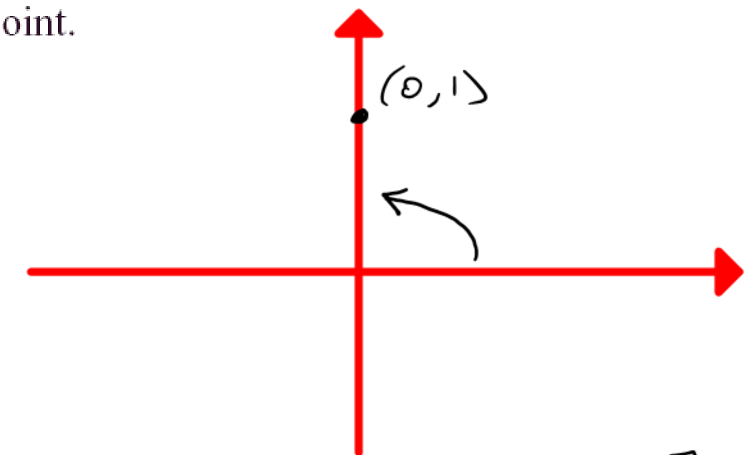
Points are specified in rectangular coordinates. Give all possible polar coordinates for each point.

$\boxed{(0,1).}$

$(-3,0).$

$(2,-2).$

$\boxed{(4\sqrt{3},4).}$



$\left[1, \frac{\pi}{2}\right], \left[1, \frac{\pi}{2}+2k\pi\right]$

$\left[-1, \frac{3\pi}{2}\right], \left[-1, \frac{3\pi}{2}+2k\pi\right]$

where k is an integer.

First Quadrant.

$$r = \sqrt{(4\sqrt{3})^2 + 4^2}$$
$$= 8$$
$$\theta = \arctan\left(\frac{4}{4\sqrt{3}}\right) = \arctan\left(\frac{1}{\sqrt{3}}\right)$$
$$= \frac{\pi}{6}$$

$\left[8, \frac{\pi}{6}\right], \left[8, \frac{\pi}{6}+2k\pi\right]$

$\left[-8, \frac{7\pi}{6}\right], \left[-8, \frac{7\pi}{6}+2k\pi\right]$

where k is an integer

Write the equation in polar coordinates:

$y = -6 \quad \longleftrightarrow \quad r\sin(\theta) = -6 \quad \text{or} \quad r = -6\csc(\theta)$

$x - 2y = 3 \qquad r\cos(\theta) - 2r\sin(\theta) = 3$

$$r = \frac{3}{\cos(\theta) - 2\sin(\theta)}$$

$x^2 + (y-1)^2 = 3 \qquad r^2\cos^2(\theta) + (r\sin(\theta) - 1)^2 = 3$

$$\underbrace{r^2\cos^2(\theta) + r^2\sin^2(\theta)}_{} - 2r\sin(\theta) + 1 = 3$$

$$\boxed{r^2 - 2r\sin(\theta) = 2}$$

Sketch the polar curve. 3 petal flower

$\boxed{r = \cos 3\theta}$

$r = \sin 2\theta$ 4 petal flower

$r = 2 + \sin\theta$

dimple

$r = -1 + 2\cos\theta$ inner loop

$r = 1 + 2\sin\theta$

inner loop

$r = 4$ circle of radius 4 centered at (0,0).

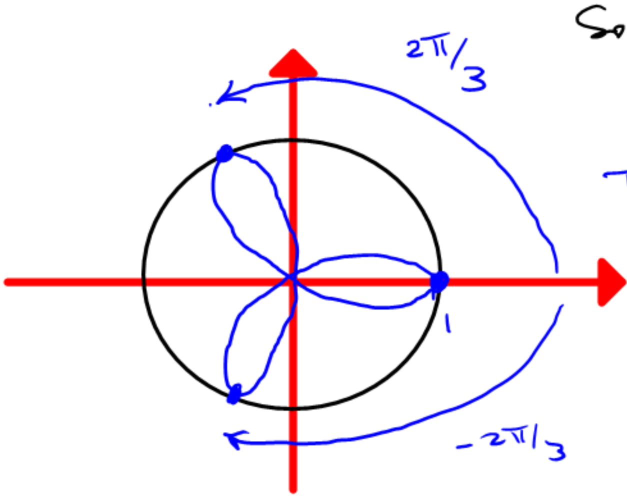
$\theta = -\frac{1}{4}\pi$ — line with slope -1 through (0,0).

petals are equally spaced.

Find the tip of one. This occurs when r is as large as possible. $r = \cos(3\theta)$

$\Rightarrow$ largest r is 1.

Solve $1 = \cos(3\theta)$. $\theta = 0$ works.

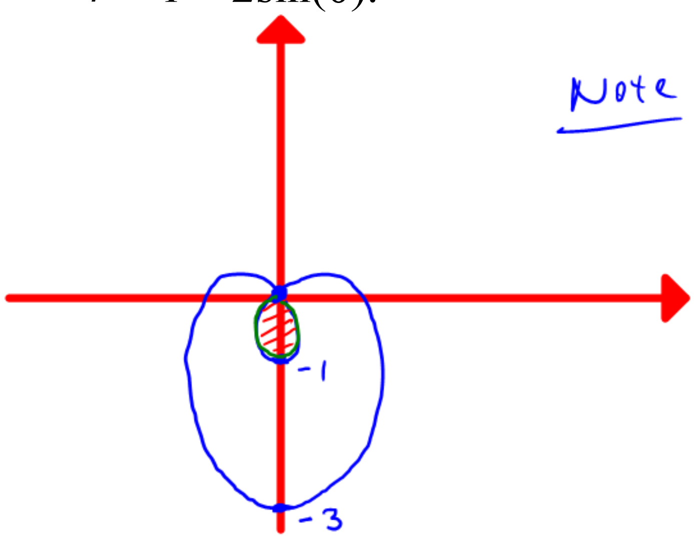


Traced twice as θ goes from 0 to 2π.

Give the area inside one petal of the polar graph $r = 2\sin(3\theta)$.

You

Give the area inside the inner loop of the polar graph
$r = 1 - 2\sin(\theta)$.



Note: The loop starts and ends at the origin.

$\underbrace{\text{the origin.}}_{r=0}$

Solve $0 = 1 - 2\sin(\theta)$

$\sin(\theta) = \frac{1}{2}$

$\theta = \pi/6\ ,\ 5\pi/6$

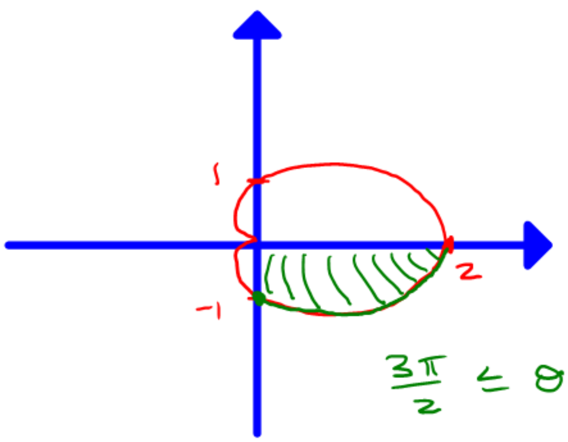
$\frac{\pi}{6} \le \theta \le \frac{5\pi}{6}$
determines the inner loop.

$$\text{Area} = \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \left(1 - 2\sin(\theta)\right)^2 d\theta$$

multiply it out and integrate.

Give the area in quadrant 4 that lies inside the polar graph
$r = 1 + \cos(\theta)$.

Cardiod.



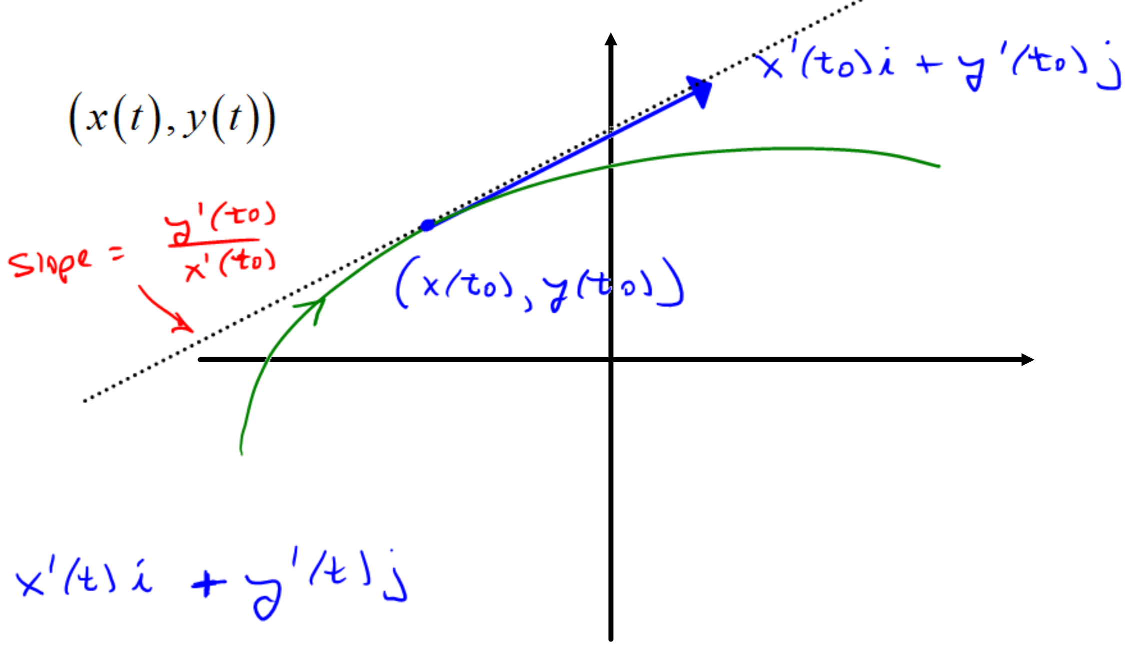
$\frac{3\pi}{2} \le \theta \le 2\pi$ to generate this piece.

$$\text{Area} = \frac{1}{2}\int_{\frac{3\pi}{2}}^{2\pi} \left(1+\cos(\theta)\right)^2 d\theta$$

...

$$\boxed{\text{Area} = \frac{1}{2}\int_a^b r(\theta)^2 d\theta}$$

Part V - Parametric Equations



Express the parametric curve by an equation in x and y.

$$\boxed{x(t) = 2t - 1, \quad y(t) = t^2 + 1}$$

$t = \frac{1}{2}(x+1)$

$\Rightarrow y = \left(\frac{1}{2}(x+1)\right)^2 + 1$

$y = \frac{1}{4}(x+1)^2 + 1$

parabola

$$\boxed{x(t) = 3\sin(t), \quad y(t) = \cos(t)}$$

$\frac{x}{3} = \sin(t) \qquad y = \cos(t)$

$\sin^2(t) + \cos^2(t) = 1$

$\Rightarrow \left(\frac{x}{3}\right)^2 + y^2 = 1$

$\frac{x^2}{9} + y^2 = 1.$

ellipse

Find a parameterization

$$x = x(t),\ \ y = y(t),\ \ t \in [0,1]$$

for the given curve.

The line segment from (3,2) to (-2,1).

The line segment from (1,3) to (0,6). ← you

The portion of the parabola $y = x^2$ from (1,1) to (-2,4).

line segment from (a, b) to (c, d)

$$\boxed{\begin{aligned} x &= a + t(c-a) \\ y &= b + t(d-b) \end{aligned}\ ,\quad 0 \le t \le 1}$$

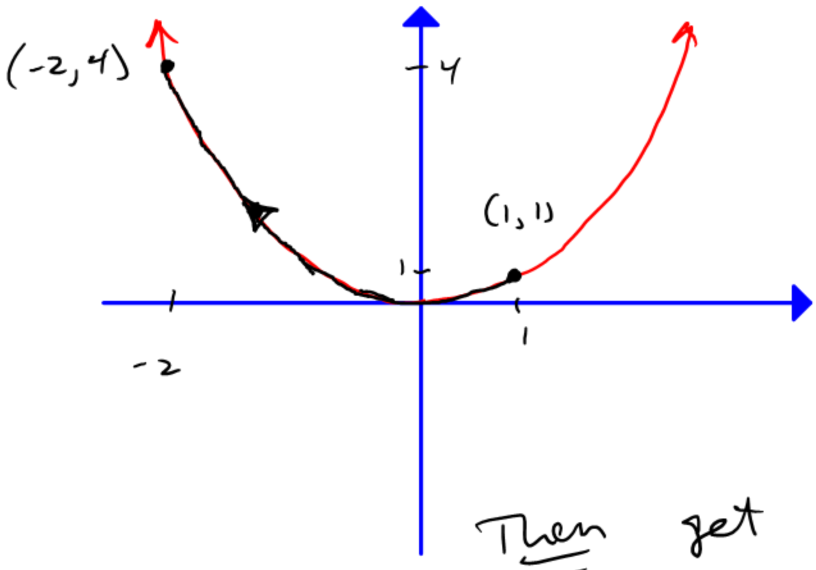
$$x = 3 + t(-2-3) \qquad 0 \le t \le 1$$

$$y = 2 + t(1-2)$$

i.e.

$$\boxed{\begin{aligned} x &= 3 - 5t \\ y &= 2 - t \end{aligned} \qquad 0 \le t \le 1}$$

The portion of $y = x^2$ from $(1,1)$ to $(-2,4)$.



First. Get ANY param for the piece.

$$\boxed{x = t,\ y = t^2 \quad -2 \le t \le 1}$$

wrong orientation

Then get the one we need.

$$\boxed{x = -t,\ y = t^2 \quad -1 \le t \le 2}$$

$-2 \le -t \le 1$

$-1 \le t \le 2$

Give tangent line and normal line to the curve at the point associated with the given value of t, using both xy equations and parametric equations.

$$x(t)=2-3\sin(t),\ y(t)=\cos(3t),\ t=\frac{\pi}{4}$$

$x'(t) = -3\cos(t)$, $y'(t) = -3\sin(3t)$

T.L.

xy equation:

$$\text{Point} = \left(x\left(\tfrac{\pi}{4}\right), y\left(\tfrac{\pi}{4}\right)\right)$$

$$= \left(2-\frac{3\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$$

$$\text{Slope} = \frac{y'(\pi/4)}{x'(\pi/4)} = \frac{-3\frac{\sqrt{2}}{2}}{-3\frac{\sqrt{2}}{2}} = 1$$

Line:

$$y - -\frac{\sqrt{2}}{2} = 1\cdot\left(x-\left(2-\frac{3\sqrt{2}}{2}\right)\right)$$

$$y = x - 2 + \sqrt{2}$$

N.L.

xy equation: Same except slope $= -1$.

$$y - -\frac{\sqrt{2}}{2} = -\left(x-\left(2-\frac{3\sqrt{2}}{2}\right)\right)$$

...

T.L. Param equation:

$$x = 2-\frac{3\sqrt{2}}{2} + t\left(-\frac{3\sqrt{2}}{2}\right)$$

$$y = -\frac{\sqrt{2}}{2} + t\left(-3\frac{\sqrt{2}}{2}\right)$$

N.L. Param:

$$x = 2-\frac{3\sqrt{2}}{2} + t\left(\frac{3\sqrt{2}}{2}\right)$$

$$y = -\frac{\sqrt{2}}{2} + t\left(-\frac{3\sqrt{2}}{2}\right)$$

T.L. at $(x(t_0), y(t_0))$

$$x = x(t_0) + t\, x'(t_0)$$

$$y = y(t_0) + t\, y'(t_0)$$

N.L. at $(x(t_0), y(t_0))$

$$x = x(t_0) + t\left(-y'(t_0)\right)$$

$$y = y(t_0) + t\left(x'(t_0)\right)$$

Find the points (x, y) at which the curve has: (a) a horizontal tangent; (b) a vertical tangent. Then sketch the curve.

$$x(t) = 3t - t^3, \quad y(t) = t + 1.$$
$$x(t) = t^2 - 2t, \quad y(t) = t^3 - 12t.$$
$$x(t) = 3 - 4\sin t, \quad y(t) = 4 + 3\cos t.$$
$$x(t) = \sin 2t, \quad y(t) = \sin t.$$
$$x(t) = t^2 - 2t, \quad y(t) = t^3 - 3t^2 + 2t.$$
$$x(t) = 2 - 5\cos t, \quad y(t) = 3 + \sin t.$$

Horiz. tangent when $y'(t) = 0$

vert tangent when $x'(t) = 0$

$x'(t) = 3 - 3t^2$

$y'(t) = 1$ No horiz tangents.

vert. tangents: $x'(t) = 0 \Rightarrow 3 - 3t^2 = 0$

$t = \pm 1.$

points: $(x(-1), y(-1))$, $(x(1), y(1))$

$(-2, 0)$, $(2, 2)$

Give a formula for the length of the curve given by

$x(t) = 2 - 3\sin(t),\ y(t) = \cos(3t)$

See other review video.

$y = x^2 - 2x,\ 1 \le x \le 3$

$r = 2 + 3\cos(\theta),\ \frac{\pi}{4} \le \theta \le \frac{3\pi}{4}$

$r'(\theta) = -3\sin(\theta)$

$$\int_{\pi/4}^{3\pi/4} \sqrt{r(\theta)^2 + r'(\theta)^2}\, d\theta$$

$$= \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \sqrt{(2+3\cos(\theta))^2 + 9\sin^2(\theta)}\, d\theta$$

The equations below give the position of a particle at each time t during the time interval specified. Find the initial speed of the particle, the terminal speed of the partical, and the distance traveled by the particle.

$$x(t) = t^2, \quad y(t) = 2t, \quad \text{from } t = 0 \text{ to } t = \sqrt{3}.$$

$$x(t) = t - 1, \quad y(t) = \tfrac{1}{2}t^2, \quad \text{from } t = 0 \text{ to } t = 1.$$

$$x(t) = t^2, \quad y(t) = t^3, \quad \text{from } t = 0 \text{ to } t = 1.$$

J04

See review #2

Part VI - Sets, Sequences, LUB, GLB, Monotonicity and Limits

Find the least upper bound (if it exists) and the greatest lower bound (if it exists) for the given set.

$\{x : x^2 < 4\}$.

$\{x : x^3 \geq 8\}$.

$\{2\frac{1}{2}, 2\frac{1}{3}, 2\frac{1}{4}, \ldots\}$.

$\{-1, -\frac{1}{2}, -\frac{1}{3}, -\frac{1}{4}, \ldots\}$.

$\{x : |x - 1| < 2\}$.

$\{x : x^4 \leq 16\}$.

$\{x : x^2 + x + 2 \geq 0\}$.

$(-2, 2)$ GLB $= -2$ LUB $= 2$

$[2, \infty)$ GLB $= 2$ LUB $=$ dne

$(-1, 3)$ GLB $= -1$ LUB $= 3$

$[-2, 2]$ GLB $= -2$ LUB $= 2$

increasing. GLB $= -1$ LUB $= 0$

$f(x)=\frac{2}{x}, \ x\geq 1$ $\qquad f'(x)=\frac{-2}{x^2}<0 \Rightarrow f$ is decreasing $\Rightarrow$ sequence is decreasing

Determine the boundedness and monotonicity of each of the sequences, and determine their limits. Also, give the LUB and GLB for the sequences marked with **RED** dots. Assume each sequence starts at $n = 1$.

- ● $\frac{2}{n}$. $\to 0$ — decreasing, bdd, GLB $= 0$, LUB $= 2$
- ● $\frac{n+(-1)^n}{n}$. $\to 1$ — bdd, GLB $= 0$, LUB $= \frac{3}{2}$, not monotone; $0, \frac{3}{2}, \frac{2}{3}, \frac{5}{4}, \frac{4}{5}, \ldots$
- ● $(0.9)^n$.
- ● $\sqrt{n^2+1}$.
- ● $\frac{2^n}{4^n+1}$.
- ● $\frac{n^2}{\sqrt{n^3+1}}$.
- $\frac{n+2}{3^{10}\sqrt{n}}$.
- $(-1)^n\sqrt{n}$.
- $\ln\left(\frac{n+1}{n}\right)$.
- ● $\frac{\sqrt{n+1}}{\sqrt{n}}$.
- ● $\frac{(-1)^n}{n}$. $\to 0$ — bdd, GLB $= -1$, LUB $= \frac{1}{2}$, not monotone; $-1, \frac{1}{2}, -\frac{1}{3}, \frac{1}{4}, \ldots$
- ● $(1.001)^n$.
- ● $\frac{n-1}{n}$.
- ● $\frac{n^2}{n+1}$. $\to \infty$ — limit dne. sequence diverges. Not bounded. But it is bounded below. Not bdd above. GLB $= \frac{1}{2}$
- $\frac{4n}{\sqrt{4n^2+1}}$.
- $\frac{4^n}{2^n+100}$.
- ● $\ln\left(\frac{2n}{n+1}\right)$.

Monotone?

$f(x)=\frac{x^2}{x+1}, \ x\geq 1$

$f'(x)=\frac{(x+1)\cdot 2x - x^2}{(x+1)^2}=\frac{2x^2+2x}{(x+1)^2}$ > 0

$\Rightarrow f$ is increasing

$\Rightarrow$ sequence is increasing

$\Rightarrow$ sequence is monotone