Test 4 Review
Spring 2012

Topics

Infinite Series:
- Convergence, divergence, absolute convergence, conditional convergence.
- Alternating series and alternating series test.
- Convergence tests for series with nonnegative terms - integral test, comparison test, limit comparison test, ratio test, root test.
- Special series (p-series, geometric series).

L'Hospital's Rule:
- Indeterminant forms.
- Applying the theorem.

Improper Integrals:
- Identification.
- Computation using proper notation.

Taylor Polynomial Approximation:
- Formula for Taylor polynomials.
- Taylor polynomials for simple functions.
- Error estimation and prediction of n to satisfy an error bound.

Practice Questions

Example: Give the value of $\sum_{n=2}^{\infty} \frac{\cos(n\pi)}{4^n}$

Example: Determine whether the series converges or diverges. Show your work.

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt[4]{n^3}}$$

Example: Determine whether the series converges or diverges. Show your work.

$$\sum_{n=1}^{\infty} \frac{2^n}{n^3}$$

Example: Determine whether the series converges or diverges. Show your work.

$$\sum_{n=1}^{\infty} \left(\frac{1}{n+1} - \frac{1}{n} \right)$$

Example: Determine whether the series converges or diverges. Show your work.

$$\sum_{n=1}^{\infty} \frac{3^{2n}}{n!}$$

Example: Determine whether the series converges or diverges. Show your work.

$$\sum_{n=1}^{\infty} \cos(\pi n)$$

Example: Determine whether the series converges or diverges. Show your work.

$$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n}$$

Example: Determine whether the series converges or diverges. Show your work.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1} n^2}{3n^3 + 1}$$

Example: Determine whether the series converges or diverges. Show your work.

$$\sum_{n=0}^{\infty} 3\left(-\frac{1}{2}\right)^n$$

Example: Determine whether the series converges or diverges. Show your work.

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$$

ex. $\sum_{n=2}^{\infty} \frac{1}{n \ln(n)}$

similar.

you do it.

Diverges.

Example: Determine whether the series converges or diverges. Show your work.

$$\sum_{n=1}^{\infty} ne^{-n^3}$$

Example: Determine whether the series converges or diverges. Show your work.

$$\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^n$$

Example: Determine whether the series converges or diverges. Show your work.

$$\sum_{n=1}^{\infty} \frac{1}{n^3+1}$$

Example: Determine whether the series converges or diverges. Show your work.

$$\sum_{n=0}^{\infty}\left(\frac{2}{9}\right)^n$$

Example: Determine whether the series converges or diverges. Show your work.

$$\sum_{n=1}^{\infty}\frac{n^2}{2^n}$$

Example: Determine whether the series converges or diverges. Show your work.

$$\sum_{n=2}^{\infty}\frac{10n^2+n-2}{2n^6+7n-1}$$

Example: Determine whether the series converges or diverges. Show your work.

$$\sum_{n=1}^{\infty}\frac{n^2+3n-2}{\sqrt{4n^9+n-1}}$$

Example: Determine whether the series converges absolutely, converges conditionally, or diverges. Show your work.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\sqrt{n}}{n+3}$$

Example: Determine whether the series converges absolutely, converges conditionally, or diverges. Show your work.

$$\sum_{n=1}^{\infty} \frac{\cos \pi n}{n^2}$$

Example: Determine whether the series converges absolutely, converges conditionally, or diverges. Show your work.

$$\sum_{n=0}^{\infty} \frac{4n(-1)^n}{3n^2+2n+1}$$

Example: Determine whether the series converges absolutely, converges conditionally, or diverges. Show your work.

$$\sum_{n=0}^{\infty} \frac{3(-1)^n}{\sqrt{3n^2+2n+1}}$$

Example: Determine whether the series converges absolutely, converges conditionally, or diverges. Show your work.

$$\sum_{n=0}^{\infty} \frac{3n(-1)^n}{\sqrt{3n^2 + 2n + 1}}$$

Example: Determine whether the series converges absolutely, converges conditionally, or diverges. Show your work.

$$\sum_{n=0}^{\infty} \left(4(-1)^n \left(\frac{n}{n+3} \right)^n \right)$$

Example: Determine whether the series converges absolutely, converges conditionally, or diverges. Show your work.

$$\sum_{n=0}^{\infty} \left(\frac{2(-1)^n \arctan n}{3 + n^2 + n^3} \right)$$

Example: Determine whether the series converges absolutely, converges conditionally, or diverges. Show your work.

$$\sum_{n=0}^{\infty} \left(\frac{(-1)^n 3^n}{4^n + 3n} \right)$$

Example: Determine whether the series converges absolutely, converges conditionally, or diverges. Show your work.

$$\sum_{n=0}^{\infty}\left(\frac{(-1)^n 3}{(n+2)\ln(n+2)}\right)$$

Example: Determine whether the series converges absolutely, converges conditionally, or diverges. Show your work.

$$\sum_{n=2}^{\infty}\frac{(-1)^n n!}{(n+1)!}$$

Example: Determine whether the series converges absolutely, converges conditionally, or diverges. Show your work.

$$\sum_{n=2}^{\infty}\frac{(-1)^n}{3n+2}$$

Example: Determine whether the series converges absolutely, converges conditionally, or diverges. Show your work.

$$\sum_{n=0}^{\infty}\frac{(-1)^n 10n^2}{3^n}$$

Example: Determine whether the series converges absolutely, converges conditionally, or diverges. Show your work.

$$\sum_{n=2}^{\infty} \frac{(-1)^n 3^n}{n!}$$

Example: Determine whether the series converges absolutely, converges conditionally, or diverges. Show your work.

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n^2 + 3n + 2}$$

Example: Determine whether the series converges absolutely, converges conditionally, or diverges. Show your work.

$$\sum_{n=2}^{\infty} \frac{\cos(\pi n) n^n}{n!}$$

Example: Determine whether the series converges absolutely, converges conditionally, or diverges. Show your work.

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln(n))^2}$$

Example: Determine whether the limit is in indeterminant form. Then compute the limit.

$$\lim_{n\to\infty}\frac{\ln(n+4)}{n+2}$$

Example: Determine whether the limit is in indeterminant form. Then compute the limit.

$$\lim_{n\to\infty}(3n)^{\frac{2}{n}}$$

Example: Determine whether the limit is in indeterminant form. Then compute the limit.

$$\lim_{n\to\infty}\left(1+\frac{3}{n}\right)^{2n}$$

Example: Determine whether the limit is in indeterminant form. Then compute the limit.

$$\lim_{x\to 0}\frac{x-\sin(2x)}{x+\sin(2x)}$$

Example: Determine whether the limit is in indeterminant form. Then compute the limit.

$$\lim_{x->0} \frac{e^{x^2} - 1}{2x^2}$$

Example: Determine whether the limit is in indeterminant form. Then compute the limit.

$$\lim_{x->0+} \left(\frac{1}{x}\right)^x$$

Example: Determine whether the limit is in indeterminant form. Then compute the limit.

$$\lim_{x->0} \frac{3e^{x/3} - (3+x)}{x^2}$$

Example: Determine whether the limit is in indeterminant form. Then compute the limit.

$$\lim_{x->\infty} \frac{x^2}{\ln x}$$

Example: Determine whether the limit is in indeterminant form. Then compute the limit.

$$\lim_{x->0} \frac{1+x-e^x}{x(e^x-1)}$$

Example: Determine whether the limit is in indeterminant form. Then compute the limit.

$$\lim_{x->0} \frac{\arctan(4x)}{x}$$

Example: Determine whether the limit is in indeterminant form. Then compute the limit.

$$\lim_{x\to\infty} \frac{\sin(2x)}{3x}$$

Example: Evaluate the limit using proper notation.

$$\int_0^{27} x^{-2/3} dx$$

Example: Evaluate the limit using proper notation.

$$\int_0^4 \frac{1}{\sqrt{4-x}}dx$$

Example: Evaluate the limit using proper notation.

$$\int_{-2}^0 \frac{1}{x+1}dx$$

Example: Evaluate the limit using proper notation.

$$\int_1^\infty \frac{1}{x^2+1}dx$$

Example: Evaluate the limit using proper notation.

$$\int_1^4 \frac{1}{x+1}dx$$

Example: Give the 4th degree Taylor polynomial centered at 0 for $\sin(x)$

Example: Give the 4th degree Taylor polynomial centered at 0 for $\cos(x)$

Example: Give the 4th degree Taylor polynomial centered at 0 for $\exp(x)$

Example: Give the 4th degree Taylor polynomial centered at 0 for $\ln(x+1)$

Example: Give the 4th degree Taylor polynomial centered at 0 for $\cos(2x)$

Example: Give the 6th degree Taylor polynomial centered at 0 for $\sin\left(x^2\right)$

Example: Write the following in powers of $(x + 1)$: $3x^3 - x^2 + 7x - 6$

Example: $f(0) = 1$, $f'(0) = 2$, $f''(0) = -4$ and $f'''(0) = 1/2$. Give the 3rd degree Taylor polynomial centered at 0.

Example: $f(-1) = -2$, $f'(-1) = 3$, $f''(-1) = 4$ and $f'''(-1) = 3/2$. Give the 3rd degree Taylor polynomial centered at -1.

Example: Give the smallest value of n so that the n^{th} degree Taylor polynomial centered at 0 approximates exp(-2) within 10^{-1}.

Example: Give the smallest value of n so that the n^{th} degree Taylor polynomial centered at $\pi/3$ approximates $\cos(70^{o})$ within 10^{-3}.