Test 4 Review
Spring 2012

Topics

Infinite Series:
• Convergence, divergence, absolute convergence, conditional convergence.
• Alternating series and alternating series test.
• Convergence tests for series with nonnegative terms - integral test, comparison test, limit comparison test, ratio test, root test.
• Special series (p-series, geometric series).

L'Hospital's Rule:
• Indeterminate forms.
• Applying the theorem.

Improper Integrals:
• Identification.
• Computation using proper notation.

Taylor Polynomial Approximation:
• Formula for Taylor polynomials.
• Taylor polynomials for simple functions.
• Error estimation and prediction of n to satisfy an error bound.

Practice Questions

Example: Give the value of \[ \sum_{n=2}^{\infty} \frac{\cos(n\pi)}{4^n} \]

Example: Determine whether the series converges or diverges. Show your work.

\[ \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \]
Example: Determine whether the series converges or diverges. Show your work.

\[ \sum_{n=1}^{\infty} \frac{2^n}{n^3} \]

Example: Determine whether the series converges or diverges. Show your work.

\[ \sum_{n=1}^{\infty} \left( \frac{1}{n+1} - \frac{1}{n} \right) \]

Example: Determine whether the series converges or diverges. Show your work.

\[ \sum_{n=1}^{\infty} \frac{3^{2n}}{n!} \]

Example: Determine whether the series converges or diverges. Show your work.

\[ \sum_{n=1}^{\infty} \cos(\pi n) \]
Example: Determine whether the series converges or diverges. Show your work.
\[
\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n}
\]

Example: Determine whether the series converges or diverges. Show your work.
\[
\sum_{n=1}^{\infty} \frac{(-1)^{n-1} n^2}{3n^3 + 1}
\]

Example: Determine whether the series converges or diverges. Show your work.
\[
\sum_{n=0}^{\infty} 3 \left( -\frac{1}{2} \right)^n
\]

Example: Determine whether the series converges or diverges. Show your work.
\[
\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^3}
\]
Example: Determine whether the series converges or diverges. Show your work.

\[ \sum_{n=1}^{\infty} ne^{-n^3} \]

Example: Determine whether the series converges or diverges. Show your work.

\[ \sum_{n=1}^{\infty} \frac{1}{n^3 + 1} \]
**Example:** Determine whether the series converges or diverges. Show your work.

\[
\sum_{n=0}^{\infty} \left( \frac{2}{9} \right)^n
\]

**Example:** Determine whether the series converges or diverges. Show your work.

\[
\sum_{n=1}^{\infty} \frac{n^2}{2^n}
\]

**Example:** Determine whether the series converges or diverges. Show your work.

\[
\sum_{n=2}^{\infty} \frac{10n^2 + n - 2}{2n^6 + 7n - 1}
\]

**Example:** Determine whether the series converges or diverges. Show your work.

\[
\sum_{n=1}^{\infty} \frac{n^2 + 3n - 2}{\sqrt[4]{4n^9} + n - 1}
\]
Example: Determine whether the series converges absolutely, converges conditionally, or diverges. Show your work.

\[ \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \sqrt{n}}{n+3} \]

Example: Determine whether the series converges absolutely, converges conditionally, or diverges. Show your work.

\[ \sum_{n=1}^{\infty} \frac{\cos \pi n}{n^2} \]

Example: Determine whether the series converges absolutely, converges conditionally, or diverges. Show your work.

\[ \sum_{n=0}^{\infty} \frac{4n(-1)^n}{3n^2 + 2n + 1} \]

Example: Determine whether the series converges absolutely, converges conditionally, or diverges. Show your work.

\[ \sum_{n=0}^{\infty} \frac{3(-1)^n}{\sqrt{3n^2 + 2n + 1}} \]
Example: Determine whether the series converges absolutely, converges conditionally, or diverges. Show your work.

\[
\sum_{n=0}^{\infty} \frac{3n(-1)^n}{\sqrt{3n^2 + 2n + 1}}
\]

Example: Determine whether the series converges absolutely, converges conditionally, or diverges. Show your work.

\[
\sum_{n=0}^{\infty} 4(-1)^n \left( \frac{n}{n+3} \right)^n
\]

Example: Determine whether the series converges absolutely, converges conditionally, or diverges. Show your work.

\[
\sum_{n=0}^{\infty} \left( \frac{2(-1)^n \arctan n}{3 + n^2 + n^3} \right)
\]

Example: Determine whether the series converges absolutely, converges conditionally, or diverges. Show your work.

\[
\sum_{n=0}^{\infty} \left( \frac{(-1)^n 3^n}{4^n + 3n} \right)
\]
**Example:** Determine whether the series converges absolutely, converges conditionally, or diverges. Show your work.

\[
\sum_{n=0}^{\infty} \frac{(-1)^n 3}{(n+2) \ln(n+2)}
\]

**Example:** Determine whether the series converges absolutely, converges conditionally, or diverges. Show your work.

\[
\sum_{n=2}^{\infty} \frac{(-1)^n n!}{(n+1)}
\]

**Example:** Determine whether the series converges absolutely, converges conditionally, or diverges. Show your work.

\[
\sum_{n=2}^{\infty} \frac{(-1)^n}{3n+2}
\]

**Example:** Determine whether the series converges absolutely, converges conditionally, or diverges. Show your work.

\[
\sum_{n=0}^{\infty} \frac{(-1)^n 10n^2}{3^n}
\]
Example: Determine whether the series converges absolutely, converges conditionally, or diverges. Show your work.

\[ \sum_{n=2}^{\infty} \frac{(-1)^n 3^n}{n!} \]

Example: Determine whether the series converges absolutely, converges conditionally, or diverges. Show your work.

\[ \sum_{n=2}^{\infty} \frac{(-1)^n}{n^2 + 3n + 2} \]

Example: Determine whether the series converges absolutely, converges conditionally, or diverges. Show your work.

\[ \sum_{n=2}^{\infty} \frac{\cos(n\pi)n^n}{n!} \]

Example: Determine whether the series converges absolutely, converges conditionally, or diverges. Show your work.

\[ \sum_{n=2}^{\infty} \frac{1}{n(n\ln(n))^2} \]
Example: Determine whether the limit is in indeterminant form. Then compute the limit.

\[
\lim_{n \to \infty} \frac{\ln(n + 4)}{n + 2}
\]

Example: Determine whether the limit is in indeterminant form. Then compute the limit.

\[
\lim_{n \to \infty} (3n)^{\frac{2}{n}}
\]

Example: Determine whether the limit is in indeterminant form. Then compute the limit.

\[
\lim_{n \to \infty} \left(1 + \frac{3}{n}\right)^{2n}
\]

Example: Determine whether the limit is in indeterminant form. Then compute the limit.

\[
\lim_{x \to 0} \frac{x - \sin(2x)}{x + \sin(2x)}
\]
Example: Determine whether the limit is in indeterminant form. Then compute the limit.

\[
\lim_{x \to 0} \frac{e^{x^2} - 1}{2x^2}
\]

Example: Determine whether the limit is in indeterminant form. Then compute the limit.

\[
\lim_{x \to 0} \left( \frac{1}{x} \right)^{1/x}
\]

Example: Determine whether the limit is in indeterminant form. Then compute the limit.

\[
\lim_{x \to 0} \frac{3e^{x/3} - (3 + x)}{x^2}
\]

Example: Determine whether the limit is in indeterminant form. Then compute the limit.

\[
\lim_{x \to \infty} \frac{x^2}{\ln x}
\]
**Example:** Determine whether the limit is in indeterminant form. Then compute the limit.

\[
\lim_{x \to 0} \frac{1 + x - e^x}{x(e^x - 1)}
\]

**Example:** Determine whether the limit is in indeterminant form. Then compute the limit.

\[
\lim_{x \to 0} \frac{\arctan(4x)}{x}
\]

**Example:** Determine whether the limit is in indeterminant form. Then compute the limit.

\[
\lim_{x \to 0} \frac{\sin(2x)}{3x}
\]

**Example:** Evaluate the limit using proper notation.

\[
\int_0^1 x^{-2/3} \, dx
\]
**Example:** Evaluate the limit using proper notation.

\[ \int_{0}^{4} \frac{1}{\sqrt{4-x}} \, dx \]

**Example:** Evaluate the limit using proper notation.

\[ \int_{2}^{4} \frac{1}{x+1} \, dx \]

**Example:** Evaluate the limit using proper notation.

\[ \int_{1}^{2} \frac{1}{x^2+1} \, dx \]

**Example:** Evaluate the limit using proper notation.

\[ \int_{0}^{2} \frac{1}{x+1} \, dx \]
Example: Give the 4th degree Taylor polynomial centered at 0 for $\sin(x)$

Example: Give the 4th degree Taylor polynomial centered at 0 for $\cos(x)$

Example: Give the 4th degree Taylor polynomial centered at 0 for $\exp(x)$

Example: Give the 4th degree Taylor polynomial centered at 0 for $\ln(x+1)$
Example: Give the 4th degree Taylor polynomial centered at 0 for \( \cos(2x) \)

Example: Give the 6th degree Taylor polynomial centered at 0 for \( \sin(x^2) \)

Example: Write the following in powers of \( (x + 1) \): \( 3x^3 - x^2 + 7x - 6 \)

Example: \( f(0) = 1, f'(0) = 2, f''(0) = -4 \) and \( f'''(0) = 1/2 \). Give the 3rd degree Taylor polynomial centered at 0.
Example: Give the smallest value of $n$ so that the $n^{th}$ degree Taylor polynomial centered at $0$ approximates $\exp(-2)$ within $10^{-3}$. 

Example: Give the smallest value of $n$ so that the $n^{th}$ degree Taylor polynomial centered at $\pi/3$ approximates $\cos(70^\circ)$ within $10^{-3}$. 

Example: $f(-1) = -2, f'(1) = 3, f''(-1) = 4$ and $f'''(-1) = 3/2$. Give the 3rd degree Taylor polynomial centered at $-1$. 