Assignment 2

Directions: Show all of your work on every problem, including the multiple choice problems. Correct answers without explanation will be marked as incorrect.

The **logistic equation** for the population p (**in thousands**) at time *t* of certain species is given by $\frac{dp}{dt} = p(2-p)$. Use the logistic equation to solve problems 1, 2 and 3 below.

1. If the initial population is 3000 (i.e. $p(0) = 3$), what is the limiting population as $t \to \infty$. Explain.
2. Can a population of 1000 ever decline to 500? (i.e. can $p(0) = 1$ ever result in $p(t) = ½$ at some positive time *t* ?) Explain.
3. Can a population of 1000 ever increase to 3000? Explain.
4. What is the slope of a solution curve to $\frac{dy}{dx} = x + \sin(y)$ as it passes through the point $\left(1, \frac{\pi}{2}\right)$?
5. What can you say about every solution curve to $\frac{dy}{dx} = x + \sin(y)$ for $x > 1$? Explain.
    a. It is increasing.
    b. It is decreasing.
    c. It is constant.
    d. None of these.
6. What can you say about a solution curve to $\frac{dy}{dx} = x + \sin(y)$ that passes through the point $(0,0)$?
    a. It has a maximum at (0,0).
    b. It has a minimum at (0,0).
    c. It has a point of inflection at (0,0).
    d. None of these.
7. The position of a particle (in cm) along the *x* axis at time *t* (in seconds) is governed by the solution of the differential equation $\frac{dx}{dt} = t^3 - x^3$. If the particle resides at $x = 1$ when $t = 2$, then what is the velocity when $t = 2$?
8. The position of a particle (in cm) along the *x* axis at time *t* (in seconds) is governed by the solution of the differential equation $\frac{dx}{dt} = t^3 - x^3$. If the particle is located at $x = 2$ when $t = 2.5$, can it reach the location $x = 1$ at any later time? (Think and/or use a direction field.)
    a. Yes
    b. No
    c. There is not enough information.
    d. None of these.
    e. None of these.

9. What can you say about the behavior of any solution to $\frac{dy}{dx} = -y$ as $x \to \infty$?

   a. It approaches 1.
   b. It approaches -1.
   c. It approaches 0.
   d. There is no limit.
   e. None of these.

10. What can you say about the solution to $\frac{dy}{dx} = x - y,\ y(0) = 1$?

   a. It stays below the line $y = x - 1$.
   b. It stays above the line $y = x - 1$.
   c. It intersects the line $y = x - 1$.
   d. None of these.

11. Use Euler's Method with a step size of $h = 0.1$ to approximate the solution to the initial value problem $\frac{dy}{dx} = x\sqrt{y},\ y(1) = 4$ at $x = 1.4$.
12. Use Improved Euler's Method with a step size of $h = 0.1$ to approximate the solution to the initial value problem $\frac{dy}{dx} = x\sqrt{y},\ y(1) = 4$ at $x = 1.4$.
13. Use Euler's Method with a fixed step size **and 4 steps** to approximate the solution to the initial value problem $\frac{dy}{dx} = y,\ y(0) = 1$ at $x = 1$.
14. Use Improved Euler's Method with a fixed step size **and 4 steps** to approximate the solution to the initial value problem $\frac{dy}{dx} = y,\ y(0) = 1$ at $x = 1$.
15. The room temperature in your office is kept constant at 70 degree Fahrenheit. Experience has taught you that the temperature of a cup of coffee brought to your office will drop from 120 degrees Fahrenheit to 100 degree Fahrenheit in 10 minutes. What should the initial temperature of your coffee be if you want it to take 20 minutes before it drops to 100 degrees Fahrenheit?