Online Math 3321

http://www.math.uh.edu/~jmorgan/Math3321online

5 min.

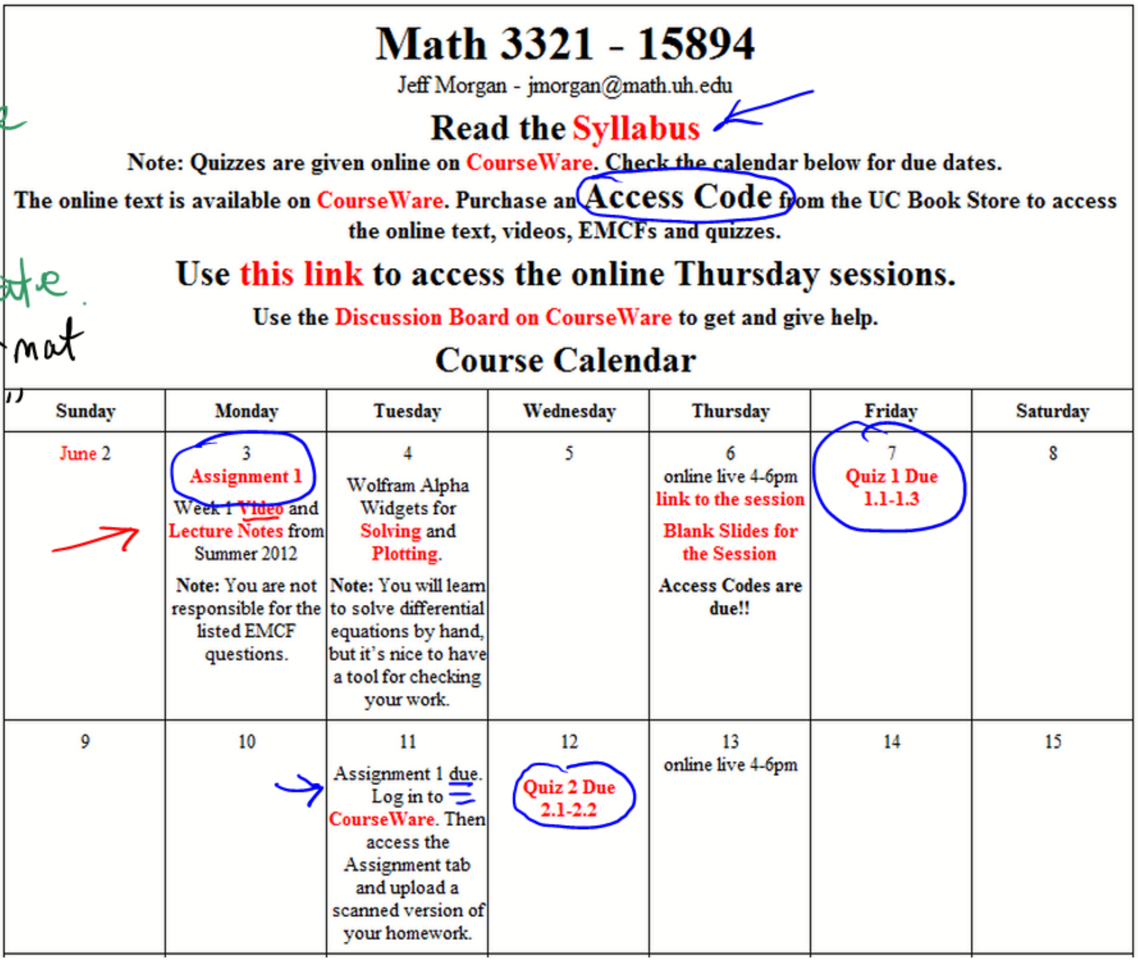
Information

- Syllabus and Course Information
- Current Assignments/Quizzes
- Discussion Board
- EMCFs - Daily Grades

CASA

every live and alternate

"mixed format EMCF"



Math 3321 - 15894

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Read the Syllabus

Note: Quizzes are given online on CourseWare. Check the calendar below for due dates.

The online text is available on CourseWare. Purchase an Access Code from the UC Book Store to access the online text, videos, EMCFs and quizzes.

Use this link to access the online Thursday sessions.

Use the Discussion Board on CourseWare to get and give help.

Course Calendar

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
June 2	3 **Assignment 1** Week 1 Video and Lecture Notes from Summer 2012 Note: You are not responsible for the listed EMCF questions.	4 Wolfram Alpha Widgets for Solving and Plotting. Note: You will learn to solve differential equations by hand, but it's nice to have a tool for checking your work.	5	6 online live 4-6pm **link to the session** **Blank Slides for the Session** **Access Codes are due!!**	7 **Quiz 1 Due 1.1-1.3**	8
9	10	11 Assignment 1 due. Log in to CourseWare. Then access the Assignment tab and upload a scanned version of your homework.	12 **Quiz 2 Due 2.1-2.2**	13 online live 4-6pm	14	15

EMCF01

1. You should log into CourseWare at http://www.casa.uh.edu during each online live session so that you can answer EMCF questions.
   (input 1 for true and 0 for false)

Today...

* **Differential Equations**
  - Definitions
  - Examples

* **Initial Value Problems**
  - Definitions
  - Examples

"ordinary differential equation"

* **Finding Solutions to Specific Types of ODEs and IVPs**
  - Separable Equations
  - Linear Equations

Definitions

order of the highest derivative that appears.

- Differential Equation and the Order of a Differential Equation

An equation for an unknown function involving one or more derivatives or partial derivatives of the function.

- Ordinary Differential Equations vs Partial Differential Equations

Diff. eq. for an unknown function of a single variable.

Diff eq for an unknown function of more than one variable.

partial derivatives will show up.

- Solution of a Differential Equation

A function that satisfies the diff. equation on a given interval.

$$f(x,y) = x^2 + 2y^3$$

$$\frac{\partial f}{\partial x} = 2x \qquad \frac{\partial f}{\partial y} = 6y^2$$

ex. $y' = 2$. ← first order ODE

← y is the unknown function.

$y = 2x$ or $y = 2x + C$ ← General sol'n.

← arbitrary constant

Mixed Format EmcF01

2. $y''' - 3y = \sin(x)$ is a __________ order differential equation. (give your answer as a number)

$\boxed{3}$

Examples of Differential Equations

$$f''(x) + 2f(x) = \frac{x}{f(x)+1}$$

2nd order $\underline{\underline{\text{ODE}}}$

f(x) is the unknown function.

$$u'(t) + (u(t)+1)^3 = \sin(t)$$

1st order $\underline{\underline{\text{ODE}}}$

u(t) is the unknown function.

$$\underline{\underline{u_{xx}}} + u_t = \sin(x+t)$$

$$u_{xx} = \frac{\partial^2 u}{\partial x^2}, \quad u_t = \frac{\partial u}{\partial t}$$

$$u \equiv u(x,t).$$

2nd order $\underbrace{\text{partial differential equation.}}_{\underline{\underline{\text{PDE}}}}$

Examples of Solutions to Differential Equations

$u' = 2$ ← the name of the ind. variable is not specified!

If nobody tells you what to use, then you can use anything you like.

Sol'ns: $u = 2x$, $u = 2t$, $u = 2z$

$u = 2x - \frac{17}{2}$, $u = 2t + 31.54$, $u = 2z - 1$

$u = 2x + C$ $\quad u = 2t + C$ $\quad u = 2z + C$

arbitrary constant

$u'(t) = 2u(t)$

The unknown function has a derivative that is twice the function.

Sol'ns: $u(t) = e^{2t}$

or $u(t) = 4e^{2t}$

or $u(t) = 2e^{2t}$

or $u(t) = C e^{2t}$

$u_{xx} + u_{yy} = 0$

or $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$

sol'ns: $u(x,y) \cancel{= \sin}(x+y).$ NOT

Can we verify this?

$u_x = \cos(x+y)$

$u_{xx} = -\sin(x+y)$

$u_y = \cos(x+y)$

$u_{yy} = -\sin(x+y)$

So $u_{xx} + u_{yy} = -2\sin(x+y)$ NOT a SOL'N

3. The function $\overset{y}{3\sin(2x)}$ is a solution to

(0) $y'' - 4y = 0$

[✓ (1)] $y'' + 4y = 0$

(2) $y'' - 2y = 0$

(3) $y'' + 2y = 0$

(4) None of these.

1

$$y' = 6\cos(2x)$$

$$\begin{array}{r} \boxed{y'' = -12\sin(2x)} \\ + \ \boxed{4y = 12\sin(2x)} \\ \hline 0 \end{array}$$

Remark: Differential equations typically have infinitely many solutions.

* We need to give some other information to pinpoint a specific solution.

← Typically "initial data"

The *general solution* to a differential equation is an expression (often involving arbitrary constants) that can be used to generate every solution of the differential equation.

IVP

Definitions

- Initial Value Problem: a differential equation, coupled with initial data...

  More specifically, an initial value problem is a k^{th} order differential equation along with the values of y, y', ... , $y^{(k-1)}$ at a given value of the independent variable.

  the same for each

ex.
$$\begin{cases} y'' + 2x\,y = \cos(x) \\ y(0) = -12 \\ y'(0) = 37.63 \end{cases}$$

$k=2$

$$u' = 2\sin(t) + \frac{t}{u+2}, \quad u(1) = 3$$

- Solution of an Initial Value Problem:

↳ A function that satisfies the differential equation and the initial data (conditions) on an interval on which the initial data is given.

Show that $y = 2e^{-3x}$ solves the IVP $\begin{cases} y' = -3y & ✓ \\ y(0) = 2 & ✓ \end{cases}$

check: $y(0) = 2e^{-3(0)} = 2$

$y' = -6e^{-3x} = -3(2e^{-3x}) = -3y$

Examples of Initial Value Problems

See the previous page.

4. $\boxed{\overbrace{y' + 2y = \sin(x + y)}^{\text{1st order ODE}},\ \overbrace{y(0) = 2}^{\text{initial data for a first order ODE}}}$

(0) is a first order differential equation

(1) is not an initial value problem

(2) is a second order initial value problem

[✓ (3)] is a first order initial value problem

(4) None of these.

$y' = 2\cos(x)$

5. The function $\overbrace{2\sin(x)}^{y}$ solves the initial value problem

~~(0) $y' + 2y = 0, y(0) = 2$~~

X (1) $y'' + 2y = 0, \underline{y(0)} = 0, \underline{y'(0)} = 2$

~~(2) $y'' + 4y = 0, y(0) = 2$~~, $y'(0) = 0$

X (3) $y'' + 4y = 0, \underline{y(0)} = 0, \underline{y'(0)} = 2$

1 [✓ (4) None of these.]

$y(0) = 0$

$y'(0) = 2$

$y'' = -2\sin(x)$

$y'' + 2y = -2\sin(x) + 4\sin(x) = 2\sin(x) \neq 0.$

Examples of Solutions to Initial Value Problems

see the prev. + video

$y'' + 4y = -2\sin(x) + 8\sin(x) = 6\sin(x) \neq 0$

Remark: $\overbrace{\text{Initial value problems}}^{\text{IVPs}}$ typically only have 1 solution.

Theorem: If x_0 and y_0 are given real numbers, and $f(x,y)$ is continuously differentiable at every point (x,y) near (x_0,y_0), then there is an open interval containing x_0 on which there is exactly one solution to the initial value problem

1st order IVP

$$\begin{cases} y' = f(x,y) \\ y(x_0) = y_0 \end{cases} \leftarrow \text{initial data}$$

The Truth: There are many differential equations for which it is IMPOSSIBLE to "write a formula" for a solution.

This does not mean there is not a solution. It just means we can't write the formula for the solution.

$\ast$

Two types that we can solve are

First Order Separable Differential Equations

and

First Order Linear Differential Equations

In fact, most USEFUL differential equations are ones for which you WILL NOT be able to write down a solution.

ex. $y' = \sin(x^2)$

ex. $y' = y^2 + x$

If we cannot write down a solution, then we can numerically approximate it (see next week). However, IT IS ALWAYS prefered to write a solution when possible since numerical approximation can sometimes lead to problems.

Algebraic equations you can't solve without approximation:

$x^2 = 2$.

$\sin(x) + x = 1$.

2 types we can TRY to solve.

First Type

$\circledast$

First Order Separable Differential Equations

$$\frac{dy}{dx} = f(x)g(y)$$

$$\frac{dz}{dt} = 2z\left(\frac{\cos(t)}{\sin(t)+3}\right)$$

Examples: $\frac{dy}{dx} = -xy$, $\frac{dy}{dx} = \frac{2x}{y+1}$, $\frac{dz}{dt} = \frac{2z\cos(t)}{\sin(t)+3}$

$$\frac{dy}{dx} = \underbrace{(-x)}_{f(x)}\underbrace{(y)}_{g(y)}$$

$$\frac{dy}{dx} = \underbrace{(2x)}_{f(x)}\underbrace{\left(\frac{1}{y+1}\right)}_{g(y)}$$

6. Is the following differential equation separable?

$$y' = \underline{-2y}\underline{e^{-2x}}$$

[✓ (0) Yes]

(1) No

7. Is the following differential equation separable?

$$y' = xy - 2e^{-2x}$$

(0) Yes

[✓ (1) No]

ex. $y' = e^{x+y} = e^x e^y$

yes

Solving Separable Differential Equations

(finding the general solution)

$$\frac{dy}{dx} = f(x)g(y)$$

1. Separate $\qquad \frac{1}{g(y)}\,dy = f(x)\,dx$

2. Integrate $\qquad \int \frac{dy}{g(y)} = \int f(x)\,dx$ ← sticky part b/c the integration might be impossible.

3. Solve for the unknown function (if possible) to obtain the form of the general solution.

(only if you can integrate)

As a result, you could have a separable differential equation, and still not be able to solve it because of the integration.

Note: If you can integrate, but you cannot solve for the unknown function, then the resulting equation is said to give the solution implicitly.

Example: Find the solution to $\overbrace{\frac{dy}{dx}=\frac{x}{y+1},\ y(0)=3}^{1^{st}\text{ order IVP.}}$

1. Solve (if possible) the ODE.

$$\frac{dy}{dx}=\frac{x}{y+1} \qquad \text{separable!}$$

$$(y+1)\,dy = x\,dx$$

$$\int (y+1)\,dy = \int x\,dx$$

Note: Both sides will have a constant of integration. We combine these as one constant on the right hand side.

$$\frac{y^2}{2}+y=\frac{x^2}{2}+C$$

$y(0)=3$ means $y=3$ when $x=0$.

$$\frac{9}{2}+3=0+C \Rightarrow C=\frac{15}{2}.$$

$$\frac{y^2}{2}+y=\frac{x^2}{2}+\frac{15}{2}.$$

It is possible to solve this for y. If you do this, you will get 2 solutions, which might confuse you since initial value problems are supposed to only have ONE. However, one of these will be extraneous.

$$\frac{y^2}{2} + y - \left(\frac{x^2}{2} + \frac{15}{2} \right) = 0$$

Quadratic in y.

Use the quadratic formula:

$$y = \frac{-1 \pm \sqrt{1 + 4 \cdot \frac{1}{2} \cdot \left(\frac{x^2}{2} + \frac{15}{2} \right)}}{1}$$

i.e. $y = -1 \pm \sqrt{16 + x^2}$

$\therefore$ $y = -1 + \sqrt{16 + x^2}$ or $y = -1 - \sqrt{16 + x^2}$

BUT recall $y(0) = 3$.

$3 = -1 + \sqrt{16}$ ✓

$3 = -1 - \sqrt{16}$ NO

$\therefore$ the sol'n is

$$y = -1 + \sqrt{16 + x^2}.$$

8. Is the following differential equation separable?

$$y' = \frac{-x}{x+y}$$

(0) Yes

[✓ (1) No]

winplot - Free Software

http://math.exeter.edu/rparris/winplot.html

$\underline{\underline{2^{nd}\text{ type}}}$

First Order Linear Differential Equations

$$\frac{dy}{dx} + \underline{\underline{p(x)}}\,y = \underline{\underline{f(x)}}$$

Illustrative Example: Find the general solution to $\underline{\frac{dy}{dx} = 3y + e^{-x}}$

$$\underbrace{\frac{dy}{dx} + \underline{\underline{(-3)}}\,y}\; = \underline{\underline{e^{-x}}}$$

Transform into the derivative of a product by multiplying both sides by an $\underbrace{\text{"integrating factor"}}_{\mu(x)}$.

$$\underbrace{\overbrace{\mu(x)\frac{dy}{dx}} + \overbrace{\overbrace{(-3)\mu(x)}\;y}} = \mu(x)e^{-x}$$

$\underline{\underline{\text{Hope}}}$ $\quad \frac{d}{dx}\left(\mu(x)\,y\right) = \mu(x)e^{-x}$

Note: $\frac{d}{dx}(\mu(x)\,y) = \underbrace{\mu(x)\frac{dy}{dx}}_{} + \underbrace{\mu'(x)\,y}_{}$.

Need $\mu'(x) = -3\mu(x)$

$\mu(x) = e^{-3x}$ works.

$$\therefore \quad \frac{d}{dx}\left(e^{-3x} y\right) = e^{-3x} e^{-x}$$

$$\frac{d}{dx}\left(e^{-3x} y\right) = e^{-4x}$$

Integrate! $\quad e^{-3x} y = -\frac{1}{4} e^{-4x} + C$

$$\Rightarrow \quad \underbrace{y = -\frac{1}{4} e^{-x} + C e^{3x}}_{\text{general sol'n to diff eq.}}$$

$\therefore$ ≡ therefore

$\ni$ ≡ such that

$\forall$ ≡ for every

$\exists$ ≡ there exists

$\because$ ≡ since

$\exists!$ ≡ there exists a unique

$\Longrightarrow$ ≡ implies

Solution Process for First Order Linear Differential Equations (how to find the general solution)

$$\frac{dy}{dx}+p(x)y=f(x)$$

Step 1: Make the left hand side look like the derivative of a product by multiplying both sides by a special function $\mu(x)$.

= a integrating factor.

$$\underbrace{\mu(x)\frac{dy}{dx}+\boxed{p(x)\mu(x)}y}=\mu(x)f(x)$$

$$\frac{d}{dx}\left(\mu(x)y\right)=\mu(x)f(x)$$

Need $\quad \mu'(x)=p(x)\mu(x)$

use $\quad \mu(x)=\exp\left(\int p(x)dx\right)$

No need for a constant of integration here, because we just need any old integrating factor that works.

Now just integrate and solve for y.

Sticky point: $\int u(x) f(x) dx$

might be problematic.

See the video posted on Monday for this part of the discussion.

Important Special Case

(of first order linear differential equations)

$$\frac{dy}{dx} = k\,y$$

In General...

$$\frac{dy}{dx} = k\,y \quad \Leftrightarrow \quad y = C\,e^{kx}$$

Note: Other independent/dependent variable names are also possible.

$$\frac{du}{dt} = -4u \quad \Leftrightarrow \quad u = C\,e^{-4t}$$

$$u'(t) = 2u(t) \quad \Leftrightarrow \quad u(t) = C\,e^{2t}$$

$$w'(z) = -3\,w(z) \quad \Leftrightarrow \quad w(z) = C\,e^{-3z}$$

Example: Solve the initial value problem $\frac{dy}{dx} = 3y + e^{-x}$, $y(0) = 2$

Example: Solve the initial value problem $\frac{dy}{dx}+xy=3x,\ y(0)=2$

Example: Give the general solution to $x\,y' + 4y = \dfrac{-2e^{3x}}{x^3}$