Midterm Exam

Not posted yet

Friday, July 12th
2-5:00pm

or

Saturday, July 13th
9am-noon

Use the online form on the course homepage to choose your date/time.

Open EMCF04

23	24 **Assignment 4** Week 4 from Summer 2012 Videos: **1**, **2** **Notes**	25 **Assignment 3 due** **Alternate 3 due** Log in to **CourseWare**. Then access the Assignment tab and upload a scanned version of your homework.	26 **Laplace Transform Videos**	27 online live 4-6pm **link to the session** **Blank Slides** **Quiz 5 Due 3.4-3.5**	28 Alternate 4	29 will be posted tomorrow.
30	July 1	2 **Quiz 6 Due 3.6, 3.7, 4.1, 4.2** **Assignment 4 due** **Alternate 4 due**	3	4 No live meeting **Read Sections 4.4 and 4.5, and watch the associated videos.**	5 **Midterm Review Problems** **Material:** Everything through section 4.3.	6
7	8 **Quiz 7 Due 4.2-4.4**	9	10	11 online live 4-6pm	12 **Midterm Exam** from 2-5:00pm. Room TBD	13 **Quiz 8 Due 4.4, 4.5, 5.1-5.3** **Midterm Exam** from 9am-noon. Room TBD

plenty in video on Monday.

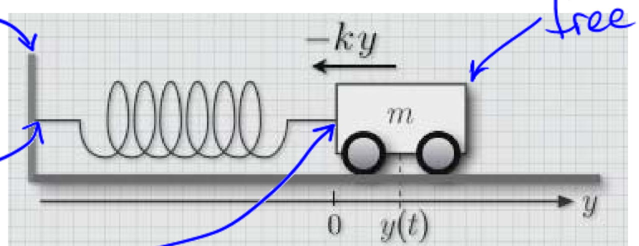
Simple Applications of Second Order Linear Differemtial Equations - Section 3.6

Spring Mass Systems

Part I: No damping and no external forces - Simple Harmonic Motion.

fixed

free to roll. No friction.



attached

approximation

Hooke's law states that for small displacements, the restoring force is proportional to the displacement.

accel.

$$m\,y'' + k\,y = 0.$$

Hooke's law.

$y(t)$

$y \equiv$ displacement

$$\omega = \sqrt{k/m}$$

balancing forces leads to a second order ODE.

Terms: Period, frequency, amplitude, phase shift.

$m > 0$

$k > 0$

read and/or watch Monday's video.

what is the period of $\cos(\sqrt{2}\,t)$? $2\pi/\sqrt{2}$

$$y'' + \frac{k}{m}\,y = 0.$$

positive constant.

$$r^2 + \frac{k}{m} = 0$$

$$r^2 = -\frac{k}{m}$$

roots $\quad r = \pm\sqrt{\frac{k}{m}}\;i$

$$\therefore \quad y = c_1 \cos\left(\sqrt{\tfrac{k}{m}}\,t\right) + c_2 \sin\left(\sqrt{\tfrac{k}{m}}\,t\right).$$

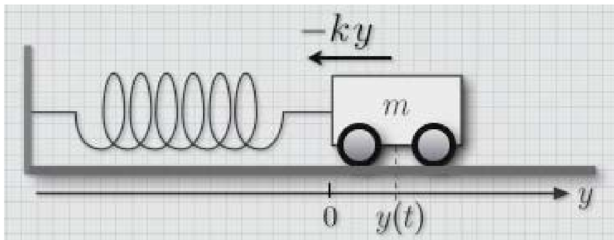
period: $\quad 2\pi/\sqrt{k/m} = 2\pi\sqrt{m/k}$

simplifying assumption:

Part II: Damping, but no external forces.

damping is proportional to speed.

balancing forces



resists motion.

cart $y' > 0$

damping

$$m\,y'' = -k\,y - \underline{\delta\, y'}$$

$\delta > 0$

Hooke's law

cart $y' < 0$

damping

$$\underbrace{m\,y'' + \delta\,y' + k\,y = 0}$$

second order constant coef ODE.

Terms: Overdamped, underdamped, critically damped.

read or watch video.

The Forced System

$$m\,y'' + \delta\,y' + k\,y = F(t)$$

external force

Special Case: $F(t) = F_0 \cos(\gamma t)$

Terms: Natural frequency = $\omega/(2\pi)$, Applied frequency = $\gamma/(2\pi)$.

read or watch

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1. 6.25

Nonhomog: $f \not\equiv 0$.

Higher Order Linear Differential Equations

$$y^{(n)} + p_{n-1}(x)y^{(n-1)} + p_{n-2}(x)y^{(n-2)} + \cdots + p_1(x)y' + p_0(x)y = f(x) \qquad (NH)$$

$$y^{(n)} + p_{n-1}(x)y^{(n-1)} + p_{n-2}(x)y^{(n-2)} + \cdots + p_1(x)y' + p_0(x)y = 0 \qquad (H)$$

$$\underbrace{L[y(x)] = y^{(n)}(x) + p_{n-1}(x)y^{(n-1)}(x) + \cdots + p_1(x)y'(x) + p_0(x)y(x)}_{\text{associated linear differential operator.}}$$

$L[u+v] = L[u] + L[v]$

$L[\alpha u] = \alpha L[u]$.

α constant

u, v function.

Terms: Nonhomogeneous equation, homogeneous equation, linear differential operator, number of solutions for the homogeneous and nonhomogeneous equations.

infinitely many.

Examples:

- $y''' + 2y'' + 3xy = \sin(x)$.
- $y^{(4)} + 3y''' - 2xy'' + y' - e^x y = \cos(2x)$

3rd order linear nonhomogeneous ODE

4th order linear nonhomogeneous ODE.

n^{th} order

Initial Value Problem

$p_k(x)$ and $f(x)$ are given

ODE $\rightarrow$ $$y^{(n)} + p_{n-1}(x)y^{(n-1)} + p_{n-2}(x)y^{(n-2)} + \cdots + p_1(x)y' + p_0(x)y = f(x);$$

initial data. $\rightarrow$ $$y(a) = \alpha_0,\ y'(a) = \alpha_1,\ \ldots,\ y^{(n-1)}(a) = \alpha_{n-1}$$

a, α_j given real numbers.

all info given at the same value of the independent variable.

Uniqueness Theorem:

$p_k(x)$

If the coefficient functions and f are continuous, then the initial value problem has a unique solution.

2. -28.32

Finding the General Solution to the Homogeneous Equation

$$y^{(n)} + p_{n-1}(x)y^{(n-1)} + p_{n-2}(x)y^{(n-2)} + \cdots + p_1(x)y' + p_0(x)y = 0 \qquad (H)$$

essential to solving (NH).

Idea: Get a fundamental set of solutions, $\{y_1, y_2, \ldots, y_n\}$ so that each y_i solves (H) and the functions are Linearly Independent.

Linear Independence? The only constants $\alpha_1, \alpha_2, \ldots, \alpha_n$ so that $\alpha_1 y_1 + \alpha_2 y_2 + \cdots + \alpha_n y_n \equiv 0$ are ones for which all α_k are ZERO.

Terms: Linear combination of solutions, linear independence, Wronskian, fundamental set of solutions.

$$W[y_1, y_2, \ldots, y_n] = \det\begin{pmatrix} y_1 & y_2 & \cdots & y_n \\ y_1' & y_2' & \cdots & y_n' \\ \vdots & \vdots & & \vdots \\ y_1^{(n-1)} & y_2^{(n-2)} & \cdots & y_n^{(n-1)} \end{pmatrix}$$

If the functions y1, y2, ..., yn solve (H), then the Wronskian is either ALWAYS zero or NEVER zero.

As in the setting of 2nd order equations, these functions are linearly independent if and only if the Wronskian is nonzero.

Special

Finding the General Solution to the *Constant Coefficient* Homogeneous Equation

$$y^{(n)} + a_{n-1}y^{(n-1)} + a_{n-2}y^{(n-2)} + \cdots + a_1 y' + a_0 y = 0 \qquad (H)$$

constants ← real constants

Process:

1. Write the characteristic equation.

$$r^n + a_{n-1} r^{n-1} + \cdots + a_1 r + a_0 = 0$$

2. Get the roots, and use them to find a fundamental set of solutions.

How? Work with each root.

Case 1: Real root c.

Subcase: c is not repeated.

$$e^{cx}$$

Subcase: c is repeated k times. $(k \geq 2)$

$$e^{cx},\ xe^{cx},\ \ldots,\ x^{k-1}e^{cx}$$

Case 2: Complex root $\alpha + i\beta$, α, β real numbers, $\beta \neq 0$.

Subcase: Not repeated.

$$e^{\alpha x}\cos(\beta x),\quad e^{\alpha x}\sin(\beta x)$$

Subcase: repeated k times.

$$e^{\alpha x}\cos(\beta x),\ e^{\alpha x}\sin(\beta x),\ xe^{\alpha x}\cos(\beta x),\ xe^{\alpha x}\sin(\beta x),\ \ldots \text{ up to } x^{k-1}.$$

Examples: Find the general solution of the differential equation.

$$y^{(4)} - 4y''' + 7y'' - 6y' + 2y = 0$$ ← 4th order

Characteristic equation:

$$r^4 - 4r^3 + 7r^2 - 6r + 2 = 0.$$

Quick observation: $r = 1$ solves.

So, $r-1$ is a factor.

$$
\begin{array}{r|l}
 & r^3 - 3r^2 + 4r - 2 \\
r-1 & r^4 - 4r^3 + 7r^2 - 6r + 2 \\
 & -(r^4 - r^3) \\
 & -3r^3 + 7r^2 - 6r + 2 \\
 & -(-3r^3 + 3r^2) \\
 & 4r^2 - 6r + 2 \\
 & -(4r^2 - 4r) \\
 & -2r + 2 \\
 & -(-2r + 2) \\
 & 0
\end{array}
$$

$$\therefore \quad (r-1)(\underbrace{r^3 - 3r^2 + 4r - 2}_{r=1 \text{ again!}}) = 0$$

$$(r-1)(r-1)(r^2 - 2r + 2) = 0$$

$$\begin{array}{r} r^2 - 2r + 2 \\ r-1 \overline{\big)\, r^3 - 3r^2 + 4r - 2} \\ -\underline{(r^3 - r^2)} \\ -2r^2 + 4r - 2 \\ -\underline{(-2r^2 + 2r)} \\ 2r - 2 \\ -\underline{(2r-2)} \\ 0 \end{array}$$

roots : 1 repeated 2 times.

$$\frac{2 \pm \sqrt{4-8}}{2} = 1 \pm i$$

Fundamental set :

$$\{ e^x , xe^x , e^x\cos(x) , e^x\sin(x) \}$$

General sol'n :

$$y = c_1 e^x + c_2 x e^x + c_3 e^x \cos(x) + c_4 e^x \sin(x)$$

where c_1 , c_2 , c_3 and c_4 are arbitrary constants.

Infinitely many sol'ns

Examples: Find the solution of the initial value problem.

$$\underbrace{y^{(4)}-4y'''+7y''-6y'+2y=0}_{\text{same as above}},\ \underbrace{y(0)=1,\ y'(0)=0,\ y''(0)=-1,\ y'''(0)=2}_{\text{initial data.}}$$

General Sol'n to the ODE.

$$y=c_1e^x+c_2xe^x+c_3e^x\cos(x)+c_4e^x\sin(x)$$

Take care of the initial data:

$y(0)=1$: $\boxed{c_1+c_3=1}$

$y'(0)=0$: 3 product rules

$$y'=c_1e^x+c_2xe^x+c_2e^x+c_3e^x(-\sin(x))+c_3e^x\cos(x)+c_4e^x\cos(x)+c_4e^x\sin(x)$$

$$\boxed{c_1+c_2+c_3+c_4=0}$$

$y''(0)=-1$:

$$y'=(c_1+c_2)e^x+c_2xe^x+(-c_3+c_4)e^x\sin(x)+(c_3+c_4)e^x\cos(x)$$

$$y''=(c_1+c_2)e^x+c_2e^x+c_2xe^x+(-c_3+c_4)e^x\sin(x)+(-c_3+c_4)e^x\cos(x)+(c_3+c_4)e^x\cos(x)-(c_3+c_4)e^x\sin(x)$$

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3. ~~Evaluate the solution to the example above at x=2~~

3.15

$\longrightarrow c_1 + c_2 + c_2 - c_3 + c_4 + c_3 + c_4 = -1$

i.e. $\boxed{c_1 + 2c_2 + 2c_4 = -1}$

$y'''(0) = 2$: $y'' = (c_1 + 2c_2)e^x + c_2 x e^x - 2c_3 e^x \sin(x) + 2c_4 e^x \cos(x)$

$$y''' = (c_1 + 2c_2)e^x + c_2 x e^x + c_2 e^x - 2c_3 e^x \sin(x) - 2c_3 e^x \cos(x) + 2c_4 e^x \cos(x) - 2c_4 e^x \sin(x)$$

$\longrightarrow c_1 + 2c_2 + c_2 - 2c_3 + 2c_4 = 2$

i.e. $\boxed{c_1 + 3c_2 - 2c_3 + 2c_4 = 2}$

In total :

$$\begin{cases} c_1 + c_3 = 1 \\ c_1 + c_2 + c_3 + c_4 = 0 \\ c_1 + 2c_2 + 2c_4 = -1 \\ c_1 + 3c_2 - 2c_3 + 2c_4 = 2 \end{cases}$$

4 equations. 4 unknowns. Solve for c_1, c_2, c_3, c_4.

The write the sol'n using these values.

Example: Find a homogeneous linear constant coefficient differential equation of least order that has the following function as a solution.

r

root of 0

$$y = 5xe^{-2x} + 4\cos(x) - 2e^{0x}$$

comes from a root of $\pm i$

factor r^2+1

comes from a root of -2 repeated 2 times.

factor $(r+2)^2$

characteristic polynomial of least order is

$$r(r^2+1)(r+2)^2$$

$$= (r^3+r)(r^2+4r+4)$$

$$= r^5 + 4r^4 + 4r^3 + r^3 + 4r^2 + 4r$$

$$= r^5 + 4r^4 + 5r^3 + 4r^2 + 4r.$$

ODE:

$$y^{(5)} + 4y^{(4)} + 5y''' + 4y'' + 4y' = 0.$$

Finding the General Solution to the Nonhomogeneous Equation

$$y^{(n)} + p_{n-1}(x)y^{(n-1)} + p_{n-2}(x)y^{(n-2)} + \cdots + p_1(x)y' + p_0(x)y = f(x)$$

$$y = \underbrace{C_1y_1(x) + C_2y_2(x) + \cdots + C_ny_n(x)}_{\text{General sol'n to the homogeneous ODE}} + \underline{\underline{z(x)}}$$

particular sol'n. (any sol'n we can find)

Term: Particular solution.

This is only possible if the ODE is fairly simple. For example, the coefficients are constant and the function f(x) is made up of special simple functions.

Finding a Particular Solution in the Constant Coefficient Case

$$y^{(n)} + a_{n-1}y^{(n-1)} + \cdots + a_1 y' + a_0 y = f(x)$$

2 methods

Term: Undetermined coefficients, variation of parameters.

guessing method

formula in terms of integrals.

If $n \geq 3$, this is extremely complicated.

Our focus.

$f(x)$ must look like a sum of terms like e^{ax}, xe^{ax}, $\ldots$, $x^k e^{ax}$, $e^{ax}\cos(bx)$, $e^{ax}\sin(bx)$, $xe^{ax}\cos(bx)$, $xe^{ax}\sin(bx)$, $\ldots$, $x^k e^{ax}\cos(bx)$, $x^k e^{ax}\sin(bx)$. Take the terms one at a time.

e.g.

$$y'' + y' - 2y = x^2\cos(x) + 3\sin(x) + e^x$$

1. $y'' + y' - 2y = 0.$

$$r^2 + r - 2 = 0$$

$$(r+2)(r-1) = 0$$

$$r = -2, \quad r = 1.$$

$$y_H = c_1 e^{-2x} + c_2 e^x$$

2. Get $z(x)$.

Guess : $z(x) = (Ax^2+Bx+C)\cos(x) + (Dx^2+Ex+F)\sin(x)$

$\qquad + G\cos(x) + H\sin(x) + Jxe^x$

Note: duplication. We don't need $G\cos(x)$, $H\sin(x)$,

Guess $\rightarrow$ $z(x) = (Ax^2+Bx+C)\cos(x) + (Dx^2+Ex+F)\sin(x)$

$\qquad + Jxe^x$

---

e.g. $y'' + y' = x + 4$

1. solve $y'' + y' = 0$

$$r^2 + r = 0$$

$$r = 0, \; r = -1$$

$$y_H = c_1 + c_2 e^{-x}$$

2. Guess $z(x)$.

Note: $x+4 = (x+4)e^{0x}$

$$z(x) = (Ax^2+Bx)e^{0x} = Ax^2 + Bx.$$

$$2A + 2Ax + B = x + 4$$

$2A = 1 \Rightarrow A = 1/2$

$2A + B = 4 \Rightarrow B = 3$

$\frac{1}{2}x^2 + 3x$

$\parallel$

Gen. Sol'n is

$c_1 + c_2 e^{-x} + \frac{1}{2}x^2 + 3x$

Example: Find the general solution to

$$y^{(4)} + 3y'' - 4y = 2\cos(x) - 3e^x + 5$$

1. Get the general sol'n to

$$y^{(4)} + 3y'' - 4y = 0$$

$$r^4 + 3r^2 - 4 = 0$$

$$(r^2+4)(r^2-1) = 0$$

$$r = \pm 2i, \quad r = \pm 1.$$

$$y_H = c_1\cos(2x) + c_2\sin(2x) + c_3e^{-x} + c_4e^x.$$

2. Get a particular sol'n.

How? Guess.

$$z(x) = A\cos(x) + B\sin(x) + \underbrace{Cxe^x}_{x \text{ b/c } e^x \text{ solved (H).}} + D$$

$$z'(x) = -A\sin(x) + B\cos(x) + Ce^x + Cxe^x$$

Subst. into the ode.

$$y^{(4)} + 3y'' - 4y = 2\cos(x) - 3e^x + 5$$

$$z''(x) = -A\cos(x) - B\sin(x) + Ce^x + Ce^x + Cxe^x$$

$$= -A\cos(x) - B\sin(x) + 2Ce^x + Cxe^x$$

$$z'''(x) = A\sin(x) - B\cos(x) + 3ce^x + cxe^x$$

$$z^{(4)}(x) = A\cos(x) + B\sin(x) + 4ce^x + cxe^x$$

$$A\cos(x) + B\sin(x) + 4ce^x + cxe^x + 3(-A\cos(x) - B\sin(x) + 2ce^x + cxe^x)$$

$$-4(A\cos(x) + B\sin(x) + cxe^x + D)$$

$$= 2\cos(x) - 3e^x + 5$$

Easy to solve: Match terms.

$\cos(x)$:	$-6A = 2$	$\Rightarrow A = -\frac{1}{3}$
$\sin(x)$:	$-6B = 0$	$\Rightarrow B = 0.$
e^x:	$10C = -3$	$\Rightarrow C = -3/10$
5:	$-4D = 5$	$\Rightarrow D = -5/4$
xe^x: $\Rightarrow$	$0 = 0$	

$$\Rightarrow z(x) = -\frac{1}{3}\cos(x) + 0\cdot\sin(x) - \frac{3}{10}xe^x - \frac{5}{4}.$$

$\therefore$ The general sol'n to

$$y^{(4)} + 3y'' - 4y = 2\cos(x) - 3e^x + 5$$

is

$$y = c_1\cos(2x) + c_2\sin(2x) + c_3e^{-x} + c_4e^x - \frac{1}{3}\cos(x) - \frac{3}{10}xe^x - \frac{5}{4}.$$

4. 4.12 5. 6.37

Laplace Transforms

Motivation: Laplace transforms can be used to turn linear constant coefficient differential equations into algebraic equations.

$\mathcal{L}$: Function of x $\longrightarrow$ Special Function of s

DEFINITION Let f be a continuous function on $[0, \infty)$. The Laplace transform of f, denoted by $\mathcal{L}[(f(x)]$, or by $F(s)$, is the function given by

function of s.

$$\mathcal{L}[f(x)] = F(s) = \int_0^\infty e^{-sx} f(x)\, dx. \tag{1}$$

The domain of F is the set of all real numbers s for which the improper integral converges.

reminder

Illustrative Examples:

$$\boxed{L\left[f(x)\right]=\int_0^{\infty} e^{-sx} f(x)\,dx}$$

$$L\left[e^{-x}\right]=\int_0^{\infty} e^{-sx} e^{-x}\,dx = \int_0^{\infty} e^{-(s+1)x}\,dx$$

$$= \lim_{c\to\infty} \int_0^{c} e^{-(s+1)x}\,dx = \lim_{c\to\infty} \left.\frac{-1}{s+1} e^{-(s+1)x}\right|_0^c$$

$$= \lim_{c\to\infty} \left[\frac{-1}{s+1} e^{-(s+1)c} + \frac{1}{s+1}\right]$$

Need $s+1>0$
$s>-1.$

$$= \frac{1}{s+1}$$

$$\boxed{L\left[e^{-x}\right] = \frac{1}{s+1}, \quad s>-1}$$

$$L\left[e^{3x}\right]=\frac{1}{s-3}, \quad s>3$$

$$L\left[e^{ax}\right]=\frac{1}{s-a}, \quad s>a.$$

$$\int_0^{\infty} e^{-sx} e^{3x}\,dx = \int_0^{\infty} e^{-(s-3)x}\,dx$$

$$= \lim_{c\to\infty} \int_0^{c} e^{-(s-3)x}\,dx = \lim_{c\to\infty} \left.\frac{-1}{s-3} e^{-(s-3)x}\right|_0^c$$

$$= \lim_{c\to\infty} \left[\frac{-1}{s-3} e^{-(s-3)c} + \frac{1}{s-3}\right]$$

Need $s-3>0$
$s>3.$

$$= \frac{1}{s-3}, \quad s>3$$

4. ~~Evaluate the Laplace Transform of exp(-3x) at s = 1.~~

Recall: $$\boxed{L\left[f(x)\right]=\int_0^{\infty} e^{-sx} f(x)\,dx}$$

As a result... Here α is a constant

1. $L\left[\alpha f(x)\right]= \int_0^{\infty} e^{-sx}\alpha f(x)\,dx = \alpha \underbrace{\int_0^{\infty} e^{-sx} f(x)\,dx}$

$= \alpha\, L\left[f(x)\right].$

2. $L\left[f(x)+g(x)\right]= \ldots = L\left[f(x)\right] + L\left[g(x)\right]$

i.e. Laplace transform is a linear operator.

1 and 2 imply the Laplace transform is a linear transformation.

$$L[\,y'(x)\,] = \int_0^\infty e^{-sx} y'(x)\,dx = e^{-sx} y(x) \Big|_0^\infty - \int_0^\infty -se^{-sx} y(x)\,dx$$

Par B: $u = e^{-sx}$ $du = -se^{-sx}$

$dv = y'(x)\,dx$ $v = y(x)$

$$= 0 - y(0) + s\,L[y(x)]$$

i.e. $L[y'] = -y(0) + s\,L[y]$.

$$L[\,y''(x)\,] = -y'(0) + s\,L[y']$$

$$= -y'(0) - sy(0) + s^2 L[y]$$

Example: Find the Laplace transform of the solution to

$$y''-2y'+2y=e^{-2x},\ y(0)=1,\ y'(0)=-1$$

We can do this directly, without finding the solution first!!

Recall: $L\left[y'(x)\right]=-y(0)+sL\left[y(x)\right]$

$$L\left[y''(x)\right]=-y'(0)-s\,y(0)+s^2L\left[y(x)\right]$$

$$L\left[y''-2y'+2y\right]=L\left[e^{-2x}\right]$$

$$L[y'']-2L[y']+2L[y]=\frac{1}{s+2}$$

(b/c L is a linear operator)

$$-y'(0)-sy(0)+s^2L[y]-2\left(-y(0)+sL[y]\right)+2L[y]=\frac{1}{s+2}$$

$y(0)=1,\ y'(0)=-1$

$$1-s+s^2L[y]+2-2sL[y]+2L[y]=\frac{1}{s+2}$$

$$\underbrace{(s^2-2s+2)}_{\text{Look!}}\underbrace{L[y]}_{\text{solve}} = -3+s+\frac{1}{s+2}$$

Characteristic polynomial!!

This will always happen!!

$$\therefore \quad L[y] = \frac{-3+s}{s^2-2s+2} + \frac{1}{(s+2)(s^2-2s+2)}$$

AMAZING!!! We can find the Laplace transform of the solution without finding the solution.

How???? B/C the Laplace transform changed our ODE into an algebraic equation that we could solve for L[y].

More Examples: $L\left[y'(x)\right] = -y(0) + sL\left[y(x)\right]$

$L[1] =$ $L\left[e^{0x}\right] = \frac{1}{s}$

$L[x] =$ $\int_0^\infty e^{-sx} x\,dx$ = parts ← see the table on the next page.

$L\left[x^2\right] =$ parts

$\frac{s}{s^2+4}$ $=$ $L\left[\cos(2x)\right] =$ Think!

Note: $\cos(2x)$ solves $\begin{cases} y'' + 4y = 0 \\ y(0) = 1 \\ y'(0) = 0 \end{cases}$

$L\left[\sin(2x)\right] =$ see next page

recall: $L\left[y''(x)\right] = -y'(0) - s\,y(0) + s^2 L\left[y(x)\right]$

$L[y'' + 4y] = L[0]$

$L[y''] + 4\,L[y] = 0$

$\underbrace{-y'(0)}_{0} \underbrace{-s\,y(0)}_{1} + s^2 L[y] + 4L[y] = 0$

$\rightarrow (s^2+4)L[y] = s$

$\Rightarrow L[y] = \frac{s}{s^2+4}$

Table of Laplace Transforms

Add others, or create your own!!

$f(x)$	$F(s) = \mathcal{L}[f(x)]$
1	$\frac{1}{s}, \quad s > 0$
$e^{\alpha x}$	$\frac{1}{s-\alpha}, \quad s > \alpha$
$\cos \beta x$	$\frac{s}{s^2+\beta^2}, \quad s > 0$
$\sin \beta x$	$\frac{\beta}{s^2+\beta^2}, \quad s > 0$
$e^{\alpha x} \cos \beta x$	$\frac{s-\alpha}{(s-\alpha)^2+\beta^2}, \quad s > \alpha$
$e^{\alpha x} \sin \beta x$	$\frac{\beta}{(s-\alpha)^2+\beta^2}, \quad s > \alpha$
$x^n, \quad n = 1, 2, \ldots$	$\frac{n!}{s^{n+1}}, \quad s > 0$
$x^n e^{rx}, \quad n = 1, 2, \ldots$	$\frac{n!}{(s-r)^{n+1}}, \quad s > r$
$x \cos \beta x$	$\frac{s^2-\beta^2}{(s^2+\beta^2)^2}, \quad s > 0$
$x \sin \beta x$	$\frac{2\beta s}{(s^2+\beta^2)^2}, \quad s > 0$

$*$ $$L[y'(x)] = -y(0) + sL[y(x)]$$

$\#$ $$L[y''(x)] = -y'(0) - s\,y(0) + s^2 L[y(x)]$$

I will provide this table on the midterm exam.

$$L[f(x)] = \int_0^\infty e^{-sx} f(x)\,dx$$ ← definition.

EMCF04

5. ~~Evaluate the Laplace Transform of x^3 at $s = 1$.~~

$$L\left[3e^{2x}\cos(3x)+x\sin(x)-3x^2\right]=$$

$$= 3\underbrace{L\left[e^{2x}\cos(3x)\right]} + L\left[x\sin(x)\right] - 3L\left[x^2\right]$$

$$L\left[e^{\alpha x}\cos\beta x\right] = \frac{s-\alpha}{(s-\alpha)^2+\beta^2},$$

$$L\left[x\sin(\beta x)\right] = \frac{2\beta s}{(s^2+\beta^2)^2}$$

$$L\left[x^2\right] = \frac{2}{s^3}$$

$$= 3\frac{s-2}{(s-2)^2+9} + \frac{2s}{(s^2+1)^2} - \frac{6}{s^3}.$$

Note: $L\,[f\,(x)\,g\,(x)]$ is generally NOT equal to $L\,[f\,(x)]\,L\,[g\,(x)]$.

Example: Use the Laplace transform to solve

$$y'' - 2y' + y = e^{-2x}, \; y(0) = 1, \; y'(0) = -1$$

(Note: We can do this easier, without Laplace transforms, but I want to illustrate the process.)

Recall: $L\left[y'(x)\right] = -y(0) + sL\left[y(x)\right]$

$$L\left[y''(x)\right] = -y'(0) - s\,y(0) + s^2 L\left[y(x)\right]$$

See video #2 from Monday.

The TRUTH!!

The Laplace Transform is typically used to solve problems of the form

$$\begin{cases} y''+ay'+by=f(t) \\ y(0)=b,\ y'(0)=m \end{cases}$$

where $f(t)$ is a piecewise defined function.

Next time:

Also, see the Laplace Transform Videos. I posted ??

Our discussion + Monday videos

= 4.1, 4.2, 4.3.