Midterm Exam

Friday, July 13th
2-5:00pm

or

Saturday, July 14th
9am-noon

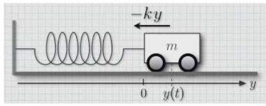
Use the online form on the course homepage to choose your date/time.

Open EMCF04b

Simple Applications of Second Order Linear Differemtial Equations - Section 3.6

Spring Mass Systems

Part I: No damping and no external forces - Simple Harmonic Motion.



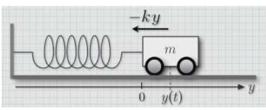
Hooke's law states that for small displacements, the restoring force is proportional to the displacement.

$$m\,y'' + k\,y = 0.$$

$$\omega = \sqrt{k/m}$$

Terms: Period, frequency, amplitude, phase shift.

Part II: Damping, but no external forces.



$$m\,y'' = -k\,y - \delta\,y'$$

$$m\,y'' + \delta\,y' + k\,y = 0$$

Terms: Overdamped, underdamped, critically damped.

The Forced System

$$m\,y'' + \delta\,y' + k\,y = F(t)$$

Special Case: $F(t) = F_0 \cos(\gamma t)$

Terms: Natural frequency = $\omega/(2\pi)$, Applied frequency = $\gamma/(2\pi)$.

Higher Order Linear Differential Equations

Nonhomogeneous if $f \neq 0$

$$y^{(n)} + p_{n-1}(x)y^{(n-1)} + p_{n-2}(x)y^{(n-2)} + \cdots + p_1(x)y' + p_0(x)y = f(x) \quad \text{(NH)}$$

$$y^{(n)} + p_{n-1}(x)y^{(n-1)} + p_{n-2}(x)y^{(n-2)} + \cdots + p_1(x)y' + p_0(x)y = 0 \quad \text{(H)}$$

homogeneous

$$L[y(x)] = y^{(n)}(x) + p_{n-1}(x)y^{(n-1)}(x) + \cdots + p_1(x)y'(x) + p_0(x)y(x)$$

Differential operator assoc. with the left hand side.

Terms: Nonhomogeneous equation, homogeneous equation, linear differential operator, number of solutions for the homogeneous and nonhomogeneous equations.

infinitely many

eg.

$$y''' - 2y'' + 3y = \sin(x)$$

$$y^{(4)} + 2y'' - xy' + e^x y = \cos(x)$$

Initial Value Problem

$$y^{(n)} + p_{n-1}(x)y^{(n-1)} + p_{n-2}(x)y^{(n-2)} + \cdots + p_1(x)y' + p_0(x)y = f(x);$$

$$\underbrace{y(a) = \alpha_0,\ y'(a) = \alpha_1,\ \ldots,\ y^{(n-1)}(a) = \alpha_{n-1}}_{n \text{ pieces of } \boxed{\text{initial data}}}$$

Uniqueness Theorem:

The initial value problem has a unique sol'n provided the coef. functions and $f(x)$ are piecewise continuous.

Finding the General Solution to the Homogeneous Equation

$$y^{(n)} + p_{n-1}(x)y^{(n-1)} + p_{n-2}(x)y^{(n-2)} + \cdots + p_1(x)y' + p_0(x)y = 0 \quad \text{(H)}$$

The general sol'n has the form

$$y = \underbrace{c_1y_1 + c_2y_2 + \cdots + c_ny_n}_{\text{linear combination of } y_1, \ldots, y_n}$$

where $c_1, \ldots, c_n$ are arbitrary constants and $y_1, \ldots, y_n$ are linearly independent solutions of (H).

Terms: Linear combination of solutions, linear independence, Wronskian, fundamental set of solutions.

$\boxed{n \text{ linearly ind. solutions}}$

Functions $y_1, y_2, \ldots, y_n$ are L.I. iff the only way that

$$c_1y_1(x) + c_2y_2(x) + \cdots + c_ny_n(x) = 0$$

for all x is if $c_1 = c_2 = \cdots = c_n = 0$.

The Wronskian of $y_1, y_2, \ldots, y_n$ is

$$W[y_1, y_2, \ldots, y_n] = \det \begin{pmatrix} y_1 & y_2 & \cdots & y_n \\ y_1' & y_2' & \cdots & y_n' \\ \vdots & \vdots & & \vdots \\ y_1^{(n-1)} & y_2^{(n-1)} & \cdots & y_n^{(n-1)} \end{pmatrix}$$

As in the setting of 2nd order equations, these functions are linearly independent if and only if the Wronskian is nonzero.

Finding the General Solution to the *Constant Coefficient* Homogeneous Equation

$$y^{(n)} + a_{n-1}y^{(n-1)} + a_{n-2}y^{(n-2)} + \cdots + a_1y' + a_0y = 0$$

Process: 1. Find the characteristic polynomial

$$r^n + a_{n-1}r^{n-1} + \cdots + a_1r + a_0$$

real numbers.

eg. $y''' - 3y'' + 2y' + y = 0$

char. poly.

$$r^3 - 3r^2 + 2r + 1$$

2. Get the roots of the characteristic polynomial AND use the roots to build n linearly independent solution.

① If a root $\tilde{r}$ is real and not repeated then we get $\boxed{e^{\tilde{r}x}}$ as one of our solutions.

② If a root $\tilde{r}$ is real and repeated (i.e. $(r-\tilde{r})^k$ is a factor of the char poly with $k>1$). Here, we get k solutions $\boxed{e^{\tilde{r}x}, xe^{\tilde{r}x}, \dots, x^k e^{\tilde{r}x}}$

③ If $\alpha \pm \beta i$ are complex roots (here α and β are real #s) then we get $e^{\alpha x}\cos(\beta x)$ and $e^{\alpha x}\sin(\beta x)$ are solutions. If these complex roots are repeated k times, we also get

$$x e^{\alpha x}\cos(\beta x), \quad x e^{\alpha x}\sin(\beta x)$$

$$\vdots \qquad\qquad \vdots$$

$$x^k e^{\alpha x}\cos(\beta x), \quad x^k e^{\alpha x}\sin(\beta x)$$

as solutions.

This process ALWAYS results in n-linearly independent sol'ns.

Examples: Find the general solution of the differential equation.

$$y^{(4)} - 4y''' + 7y'' - 6y' + 2y = 0$$

4th order, linear, constant coef., homogeneous diff eq.

1. Find the roots of the char. poly. i.e. solve

$$r^4 - 4r^3 + 7r^2 - 6r + 2 = 0$$

Note: $r=1$ is a root.

$$\begin{array}{r} r^3 - 3r^2 + 4r - 2 \\ r-1 \overline{)\, r^4 - 4r^3 + 7r^2 - 6r + 2} \\ -(r^4 - r^3) \\ \hline -3r^3 + 7r^2 - 6r + 2 \\ -(-3r^3 + 3r^2) \\ \hline 4r^2 - 6r + 2 \\ -(4r^2 - 4r) \\ \hline -2r + 2 \\ -(-2r+2) \\ \hline 0 \end{array}$$

$\Rightarrow$ Above factors to

$$(r-1)\underbrace{(r^3 - 3r^2 + 4r - 2)}_{r=1 \text{ is a root}} = 0$$

$$\begin{array}{r} r^2 - 2r + 2 \\ r-1 \overline{)\, r^3 - 3r^2 + 4r - 2} \\ -(r^3 - r^2) \\ \hline -2r^2 + 4r - 2 \\ -(-2r^2 + 2r) \\ \hline 2r - 2 \\ -(2r-2) \\ \hline 0 \end{array}$$

$$(r-1)(r-1)(r^2 - 2r + 2) = 0$$

$$(r-1)^2(r^2 - 2r + 2) = 0$$

Note: $r^2 - 2r + 2 = 0$ iff

$$r = \frac{2 \pm \sqrt{4-8}}{2} = \frac{2 \pm 2i}{2} = 1 \pm i.$$

roots: 1 repeated 2 times AND $1 \pm i$.

L.I. sol'ns: $e^x, xe^x, e^x\cos(x), e^x\sin(x)$

$\therefore$ the general sol'n is

$$c_1 e^x + c_2 x e^x + c_3 e^x \cos(x) + c_4 e^x \sin(x)$$

where c_1, c_2, c_3, c_4 are arbitrary constants.

Examples: Find the solution of the initial value problem.

$$\underbrace{y^{(4)} - 4y''' + 7y'' - 6y' + 2y = 0}_{\text{we just saw this.}}, \ \underbrace{y(0) = 1,\ y'(0) = 0,\ y''(0) = -1,\ y'''(0) = 2}_{\text{initial data}}$$

$$y = c_1 e^x + c_2 x e^x + c_3 e^x \cos(x) + c_4 e^x \sin(x)$$

is the general sol'n.

Use the initial data. We need y', y'', y'''.

$$y' = c_1 e^x + c_2 e^x + c_2 x e^x + c_3 e^x \cos(x) - c_3 e^x \sin(x) + c_4 e^x \sin(x) + c_4 e^x \cos(x)$$

$$y' = (c_1 + c_2)e^x + c_2 x e^x + (c_3 + c_4)e^x \cos(x) + (-c_3 + c_4)e^x \sin(x)$$

$$y'' = (c_1 + 2c_2)e^x + c_2 x e^x + 2c_4 e^x \cos(x) - 2c_3 e^x \sin(x)$$

$$y''' = (c_1 + 3c_2)e^x + c_2 x e^x + (2c_4 - 2c_3)e^x \cos(x) + (-2c_4 - 2c_3)e^x \sin(x)$$

$$y = c_1 e^x + c_2 x e^x + c_3 e^x \cos(x) + c_4 e^x \sin(x)$$

$$y' = (c_1 + c_2)e^x + c_2 x e^x + (c_3 + c_4)e^x \cos(x) + (-c_3 + c_4)e^x \sin(x)$$

$$y'' = (c_1 + 2c_2)e^x + c_2 x e^x + 2c_4 e^x \cos(x) - 2c_3 e^x \sin(x)$$

$$y''' = (c_1 + 3c_2)e^x + c_2 x e^x + (2c_4 - 2c_3)e^x \cos(x) + (-2c_4 - 2c_3)e^x \sin(x)$$

$$y(0) = 1,\ y'(0) = 0,\ y''(0) = -1,\ y'''(0) = 2$$

$$\left.\begin{array}{l} \boxed{c_1 + c_3 = 1} \\ c_1 + c_2 + c_3 + c_4 = 0 \\ c_1 + 2c_2 + 2c_4 = -1 \\ c_1 + 3c_2 + 2c_4 - 2c_3 = 2 \end{array}\right\} \text{4 equations with 4 unknowns}$$

$$\begin{array}{l} -2(1 + c_2 + c_4 = 0) \\ +\,(c_1 + 2c_2 + 2c_4 = -1) \\ \hline -2 + c_1 = -1 \Rightarrow c_1 = 1 \rightarrow c_3 = 0 \end{array}$$

Now use 2nd + 4th equations.

$$\left.\begin{array}{l} 1 + c_2 + c_4 = 0 \\ 1 + 3c_2 + 2c_4 = 2 \end{array}\right\} \longleftrightarrow \begin{array}{l} -3(c_2 + c_4 = -1) \\ +\,(3c_2 + 2c_4 = 1) \\ \hline -c_4 = 4 \\ c_4 = -4 \\ c_2 = 3 \end{array}$$

$$\boxed{\begin{array}{c} c_1 = 1,\ c_2 = 3,\ c_3 = 0 \\ c_4 = -4 \end{array}}$$

$\therefore$ Our solution is

$$y = c_1 e^x + c_2 x e^x + c_3 e^x \cos(x) + c_4 e^x \sin(x)$$

with $\boxed{c_1 = 1,\ c_2 = 3,\ c_3 = 0,\ c_4 = -4}$

$$\Rightarrow\ y = e^x + 3xe^x - 4e^x \sin(x).$$

Example: Find a homogeneous linear constant coefficient differential equation of least order that has the following function as a solution.

$$y = 5xe^{-2x} + 4\cos(x) - 2$$

Pieces	roots must be
$5xe^{-2x}$	-2 repeated 2 times
$4\cos(x)$	$\pm i$
-2	0

Note: $-2 = -2e^{0x}$

$\therefore$ char poly has factors $(r+2)^2$, (r^2+1), r

$$\begin{aligned}\therefore \text{char poly} &= (r+2)^2(r^2+1)r \\ &= (r^2+4r+4)(r^2+1)r \\ &= (r^2+4r+4)(r^3+r) \\ &= r^5 + r^3 + 4r^4 + 4r^2 + 4r^3 + 4r \\ &= r^5 + 4r^4 + 5r^3 + 4r^2 + 4r\end{aligned}$$

$\therefore$ the diff equation is

$$y^{(5)} + 4y^{(4)} + 5y^{(3)} + 4y'' + 4y' = 0.$$

Finding the General Solution to the Nonhomogeneous Equation

$$y^{(n)} + p_{n-1}(x)y^{(n-1)} + p_{n-2}(x)y^{(n-2)} + \cdots + p_1(x)y' + p_0(x)y = f(x) \quad (NH)$$

$$y = \underbrace{C_1 y_1(x) + C_2 y_2(x) + \cdots + C_n y_n(x)}_{\text{General solution of the homog. problem}} + \underbrace{z(x)}_{\text{particular soln}}$$

Term: Particular solution. $\leftarrow$ any soln to (NH).

Finding a Particular Solution in the Constant Coefficient Case

$$y^{(n)} + a_{n-1}y^{(n-1)} + \cdots + a_1 y' + a_0 y = f(x)$$

only case you have a chance of "hand" solving.

Even here, $f(x)$ needs to have a special form. Otherwise, the hand computations will be impossible.

Term: Undetermined coefficients, variation of parameters.

2 methods

Guessing method

formula method

BUT if $n \geq 3$ it is nasty!

need $f(x)$ to be made from sums and products of constants, x, e^{kx}, $\cos(\mu x)$, $\sin(\mu x)$.

Example: Find the general solution to

$$y^{(4)}+3y''-4y=2\cos(x)-3e^{x}+5$$

1. (important) Solve (H) first!!

$$y^{(4)}+3y''-4y=0 \qquad (H)$$

char poly: r^4+3r^2-4

Set $=0$. $r^4+3r^2-4=0$

$$(r^2+4)(r^2-1)=0$$

$$(r^2+4)(r-1)(r+1)=0$$

$\Rightarrow$ roots: $\pm 2i, 1, -1$

$\therefore$ the general sol'n to (H) is

$$y_H=c_1e^{-x}+c_2e^{x}+c_3\cos(2x)+c_4\sin(2x)$$

$$y_H=c_1e^{-x}+c_2e^{x}+c_3\cos(2x)+c_4\sin(2x)$$

$$y^{(4)}+3y''-4y=2\cos(x)-3e^{x}+5$$

2. Guess – Take advantage of linearity.

Terms on RHS		Guess
(i) $2\cos(x)$	$\leftrightarrow$	$\boxed{A\cos(x)+B\sin(x)}$
(ii) $-3e^{x}$	$\leftrightarrow$	I would guess Ae^x BUT e^x solves (H). So, I'll use $\boxed{Axe^x}$
(iii) 5	$\leftrightarrow$	$\boxed{A}$

(i) $\to$ Substitute $^*y=A\cos(x)+B\sin(x)$ into

$$y^{(4)}+3y''-4y=2\cos(x)$$

$y'=-A\sin(x)+B\cos(x)$

$*\ y''=-A\cos(x)-B\sin(x)$

$y'''=A\sin(x)-B\cos(x)$

$*\ y^{(4)}=A\cos(x)+B\sin(x)$

$$A\cos(x)+B\sin(x)+3(-A\cos(x)-B\sin(x))-4(A\cos(x)+B\sin(x))=2\cos(x)$$

$$\underbrace{(A-3A-4A)}_{2}\cos(x)+\underbrace{(B-3B-4B)}_{0}\sin(x)=2\cos(x)$$

$A=-\frac{1}{3}$ $\quad$ $B=0$

$\therefore -\frac{1}{3}\cos(x)$ "covers" the $2\cos(x)$ on the RHS.

(ii) $y^{(4)}+3y''-4y=-3e^{x}$

Subst. $y=Axe^x$ (different A)

$y'=Ae^x+Axe^x$

$y''=2Ae^x+Axe^x$

$y'''=3Ae^x+Axe^x$

$y^{(4)}=4Ae^x+Axe^x$

$$4Ae^x+Axe^x+3(2Ae^x+Axe^x)-4Axe^x=-3e^x$$

$$10Ae^x=-3e^x$$

$$\Rightarrow A=-\frac{3}{10}$$

$\therefore -\frac{3}{10}xe^x$ "covers" $-3e^x$ on the RHS

(iii) Subst $y=A$ (different A) into

$$y^{(4)}+3y''-4y=5$$

$$-4A=5 \Rightarrow A=-\frac{5}{4}$$

$\Rightarrow -\frac{5}{4}$ "covers" 5 on the RHS.

$\therefore Z=$ (Part from (i)) + (Part from (ii)) + (Part from (iii))

$$=-\frac{1}{3}\cos(x)-\frac{3}{10}xe^x-\frac{5}{4}$$

Finally, the general sol'n to

$$y^{(4)}+3y''-4y=2\cos(x)-3e^{x}+5$$

is

$$y=y_H+Z$$

$$=c_1e^{-x}+c_2e^{x}+c_3\cos(2x)+c_4\sin(2x)-\frac{1}{3}\cos(x)-\frac{3}{10}xe^x-\frac{5}{4}$$

Laplace Transforms

1. A

Motivation: Laplace transforms can be used to turn linear constant coefficient differential equations into algebraic equations.

L : functions $\longrightarrow$ new functions

DEFINITION Let f be a continuous function on $[0, \infty)$. The Laplace transform of f, denoted by $\mathcal{L}[(f(x)]$, or by $F(s)$, is the function given by

$$\mathcal{L}[f(x)] = F(s) = \int_0^\infty e^{-sx} f(x)\, dx. \qquad (1)$$

The domain of F is the set of all real numbers s for which the improper integral converges.

Illustrative Examples: $\boxed{L[f(x)] = \int_0^\infty e^{-sx} f(x)\, dx}$

$$L\left[e^{-x}\right] = \int_0^\infty e^{-sx} e^{-x}\, dx = \int_0^\infty e^{-(s+1)x}\, dx$$

$$= -\frac{1}{s+1} e^{-(s+1)x} \Big|_{x=0}^{x=\infty}$$

$$= 0 - -\frac{1}{s+1} e^{0}, \quad s > -1$$

$$= \frac{1}{s+1}, \quad s > -1$$

$$L\left[e^{3x}\right] = \frac{1}{s-3}, \quad s > 3$$

$$L\left[e^{ax}\right] = \frac{1}{s-a}, \quad s > a$$

$$L\left[e^{4x}\right] = \frac{1}{s-4}, \quad s > 4$$

$$L\left[e^{-2x}\right] = \frac{1}{s+2}, \quad s > -2$$

$$L[1] = L\left[e^{0x}\right] = \frac{1}{s}, \quad s > 0$$

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2. $L\left[e^{-3x}\right] = \dfrac{1}{s - -3}, \quad s > -3$

a. $\dfrac{1}{s-3}$, $s > 3$

b. $\dfrac{1}{s+3}$, $s > 3$

c. $\dfrac{1}{s+3}$, $s > -3$

d. $\dfrac{1}{s-3}$, $s > -3$

e. None of these.

i.e. $\dfrac{1}{s+3}, \quad s > -3$

Recall: $\boxed{L[f(x)] = \int_0^\infty e^{-sx} f(x)\, dx}$

As a result... α is a constant

1. $L[\alpha f(x)] = \int_0^\infty e^{-sx} \alpha f(x)\, dx$

$= \alpha \int_0^\infty e^{-sx} f(x)\, dx = \alpha L[f(x)]$

2. $L[f(x) + g(x)] = L[f(x)] + L[g(x)]$.

you

3. $L[f(x) - g(x)] = L[f(x)] - L[g(x)]$

1 and 2 imply the Laplace transform is a linear transformation.

ex. $L\left[2e^{-3x} - e^{5x}\right]$

$= 2L\left[e^{-3x}\right] - L\left[e^{5x}\right]$

$s > -3$
$s > 5$

$= 2 \cdot \dfrac{1}{s+3} - \dfrac{1}{s-5}, \quad s > 5$

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3. $L\left[3e^{-2x}\right]=$

a. $\dfrac{3}{s-2},\ s>2$

b $\dfrac{2}{s+3},\ s>3$

c $\dfrac{2}{s+3},\ s>-3$

d $\dfrac{3}{s+2},\ s>-2$

e. None of these.

4. $L\left[3e^{-2x}+2e^{x}-1\right]=$

a. $\dfrac{2}{s+3}+\dfrac{2}{s+1}-s,\ s>0$

b $\dfrac{2}{s+3}+\dfrac{2}{s-1}-s,\ s>3$

c $\dfrac{3}{s+2}+\dfrac{2}{s-1}+s,\ s>0$

d $\dfrac{3}{s+2}+\dfrac{2}{s+1}-\dfrac{1}{s},\ s>-2$

e. None of these.

$1=e^{0x}$

$$= 3L[e^{-2x}]+2L[e^{x}]-L[1]$$

$s>-2$
$s>1$
$s>0$

$$= 3\cdot\frac{1}{s+2}+2\cdot\frac{1}{s-1}-\frac{1}{s},\quad s>1$$

5. B

$u=e^{-sx}$ $\quad du=-se^{-sx}dx$
$dv=y'(x)dx$ $\quad v=y(x)$

$$L[y'(x)]=\int_0^\infty e^{-sx}y'(x)\,dx$$
$$=e^{-sx}y(x)\Big|_0^\infty-\int_0^\infty -se^{-sx}y(x)\,dx$$

for $s>0$, if large & y does not grow too fast

$$=0-y(0)+s\int_0^\infty e^{-sx}y(x)\,dx$$
$$=-y(0)+sL[y(x)].$$

$$L[y''(x)]=-y'(0)+sL[y'(x)]$$
$$=-y'(0)+s\left[-y(0)+sL[y(x)]\right]$$
$$=-y'(0)-sy(0)+s^2L[y(x)]$$

Example: Find the Laplace transform of the solution to

$$\boxed{y''-2y'+2y=e^{-2x}},\ y(0)=1,\ y'(0)=-1$$

We can do this directly, without finding the solution first!!

Recall: $L\left[y'(x)\right]=-y(0)+sL\left[y(x)\right]$

$L\left[y''(x)\right]=-y'(0)-s\,y(0)+s^2L\left[y(x)\right]$

Take the LT of the diff eqn.

$$L[y''-2y'+2y]=L[e^{-2x}]$$

use linearity

$$L[y'']-2L[y']+2L[y]=\frac{1}{s+2}$$

$$-y'(0)-sy(0)+s^2L[y]-2\left(-y(0)+sL[y]\right)+2L[y]=\frac{1}{s+2}$$

use $y(0)=1,\ y'(0)=-1$

$$1-s+2+L[y]\left(s^2-2s+2\right)=\frac{1}{s+2}$$

char poly for $y''-2y'+2y=0$

$$L[y](s^2-2s+2)=-3+s+\frac{1}{s+2}$$

$$\Rightarrow L[y]=\frac{-3+s}{s^2-2s+2}+\frac{1}{(s+2)(s^2-2s+2)}$$

Amazing!!! We can get the Laplace transform of the solution without knowing the solution.

More Examples: $L\left[y'(x)\right]=-y(0)+sL\left[y(x)\right]$

$L[1]=\dfrac{1}{s},\ s>0$

$L[x]=\dfrac{1}{s^2},\ s>0$

$L\left[x^2\right]=\dfrac{2}{s^3},\ s>0$

$L\left[\cos(2x)\right]=$
$L\left[\sin(2x)\right]=$ work harder.

$y=x$

$L[1]=-0+sL[x]$
$\frac{1}{s}$ $\Rightarrow L[x]=\frac{1}{s^2}$

$y=x^2$

$L[2x]=-0+sL[x^2]$
$\frac{2}{s^2}=sL[x^2]$
$\Rightarrow L[x^2]=\frac{2}{s^3}$

I did $L[\cos(x)]$ wed. night. See the video.

Table of Laplace Transforms Add others, or create your own!!

$f(x)$	$F(s) = \mathcal{L}[f(x)]$
1	$\frac{1}{s}, \quad s > 0$
$e^{\alpha x}$	$\frac{1}{s-\alpha}, \quad s > \alpha$
$\cos \beta x$	$\frac{s}{s^2+\beta^2}, \quad s > 0$
$\sin \beta x$	$\frac{\beta}{s^2+\beta^2}, \quad s > 0$
$e^{\alpha x} \cos \beta x$	$\frac{s-\alpha}{(s-\alpha)^2+\beta^2}, \quad s > \alpha$
$e^{\alpha x} \sin \beta x$	$\frac{\beta}{(s-\alpha)^2+\beta^2}, \quad s > \alpha$
$x^n, \quad n = 1, 2, \ldots$	$\frac{n!}{s^{n+1}}, \quad s > 0$
$x^n e^{rx}, \quad n = 1, 2, \ldots$	$\frac{n!}{(s-r)^{n+1}}, \quad s > r$
$x \cos \beta x$	$\frac{s^2-\beta^2}{(s^2+\beta^2)^2}, \quad s > 0$
$x \sin \beta x$	$\frac{2\beta s}{(s^2+\beta^2)^2}, \quad s > 0$

$L[y'(x)] = -y(0) + sL[y(x)]$

$L[y''(x)] = -y'(0) - s\,y(0) + s^2 L[y(x)]$

I will provide this table on the midterm exam.

$L[f(x)] = \int_0^\infty e^{-sx} f(x)\,dx$

$$L\left[3e^{2x}\cos(3x) + x\sin(x) - 3x^2\right] =$$

$$= 3L\left[e^{2x}\cos(3x)\right] + L\left[x\sin(x)\right] - 3L\left[x^2\right]$$

$$= 3 \cdot \frac{s-2}{(s-2)^2+9} + \frac{2s}{(s^2+1)^2} - 3 \cdot \frac{2}{s^3}$$

$$= \frac{3(s-2)}{(s-2)^2+9} + \frac{2s}{(s^2+1)^2} - \frac{6}{s^3}$$

$L\left[2\sin(x) - 3e^{-x} + 1\right] =$ Last night

Note: $L[f(x)\,g(x)]$ is generally NOT equal to $L[f(x)]\,L[g(x)]$.

See the extra video from wednesday. I work an example showing how to solve an IVP using the L.T.. Also see the past examples for Chapter 4.

Example: Use the Laplace transform to solve

$$y'' - 2y' + y = e^{-2x}, \; y(0) = 1, \; y'(0) = -1$$

(Note: We can do this easier, without Laplace transforms, but I want to illustrate the process.)

Recall: $L[y'(x)] = -y(0) + sL[y(x)]$

$$L[y''(x)] = -y'(0) - s\,y(0) + s^2 L[y(x)]$$

The TRUTH!!

The Laplace Transform is typically used to solve problems of the form

$$\begin{cases} y''+ay'+by=f(t) \\ y(0)=b,\ y'(0)=m \end{cases}$$

where $f(t)$ is a piecewise defined function.