Information

- The final exam dates/times are posted. The scheduler will be posted by tomorrow night.
- A practice problem set is posted.
- The final exam is comprehensive, but the latter material is weighted more heavily.
- Laplace transforms will be on the final exam, and I will provide a formula sheet.
- Assignment 8 is the last homework assignment.
- Take care of your online quizzes.
- This is the last online class meeting.

One more example with repeated eigenvalues:

Solve $\underbrace{u' = A\,u}$ with

initial value problem

$$A = \begin{pmatrix} -3 & 1 & -1 \\ -7 & 5 & -1 \\ -6 & 6 & -2 \end{pmatrix}, \quad u(0) = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$

Linear system of ODEs.

1. Find the eigen-info for A.

$$\det(A - \lambda I) = \det \begin{pmatrix} -3-\lambda & 1 & -1 \\ -7 & 5-\lambda & -1 \\ -6 & 6 & -2-\lambda \end{pmatrix}$$

$$= (-3-\lambda)\left[(5-\lambda)(-2-\lambda)+6\right] - 1\cdot\left[-7(-2-\lambda)-6\right]$$
$$-1\cdot\left[-7\cdot 6 + 6(5-\lambda)\right]$$

$$= -(3+\lambda)\left[-10-3\lambda+\lambda^2+6\right] + \left[-14-7\lambda+6\right]$$
$$-\left[-42+30-6\lambda\right]$$

$$= -(3+\lambda)\left[\lambda^2-3\lambda-4\right] -8-7\lambda+12+6\lambda$$

$$= -(3+\lambda)(\lambda^2-3\lambda-4) + 4 - \lambda$$

$$= -(3+\lambda)(\lambda-4)(\lambda+1) - (\lambda-4)$$

$$= -(\lambda-4)\left[(\lambda+3)(\lambda+1) + 1\right] = -(\lambda-4)\left[\lambda^2+4\lambda+4\right]$$
$$= -(\lambda-4)(\lambda+2)^2$$

Setting $\det(A-\lambda I)=0$ gives

$$A=\begin{pmatrix}-3 & 1 & -1\\ -7 & 5 & -1\\ -6 & 6 & -2\end{pmatrix}$$

$$-(\lambda-4)(\lambda+2)^2=0$$

Eigen values: $\lambda=4$ or $\lambda=-2$ repeated

$\lambda=4$: Find nonzero $\vec{v}$ so that $A\vec{v}=4\vec{v}$.

i.e. $(A-4I)\vec{v}=\vec{0}$

$$\begin{pmatrix}-7 & 1 & -1\\ -7 & 1 & -1\\ -6 & 6 & -6\end{pmatrix}\vec{v}=\begin{pmatrix}0\\0\\0\end{pmatrix}$$

$$\left(\begin{array}{ccc|c}-7 & 1 & -1 & 0\\ -7 & 1 & -1 & 0\\ -6 & 6 & -6 & 0\end{array}\right)\rightarrow\left(\begin{array}{ccc|c}-7 & 1 & -1 & 0\\ -1 & 1 & -1 & 0\\ 0 & 0 & 0 & 0\end{array}\right)$$

$$R_1-7R_2\rightarrow R_2 \quad \rightarrow \left(\begin{array}{ccc|c}-7 & 1 & -1 & 0\\ 0 & -6 & 6 & 0\\ 0 & 0 & 0 & 0\end{array}\right)$$

$$\rightarrow\left(\begin{array}{ccc|c}-7 & 1 & -1 & 0\\ 0 & -1 & 1 & 0\\ 0 & 0 & 0 & 0\end{array}\right)$$

$$\vec{v}=\begin{pmatrix}v_1\\v_2\\v_3\end{pmatrix} \qquad v_2=v_3 \qquad -7v_1=v_2-v_3=0 \Rightarrow v_1=0$$

$$=\begin{pmatrix}0\\v_3\\v_3\end{pmatrix}=v_3\begin{pmatrix}0\\1\\1\end{pmatrix},\quad v_3\neq 0$$

$\therefore$ For $\lambda=4$ we get nonzero multiples of $\begin{pmatrix}0\\1\\1\end{pmatrix}$.

$\lambda = -2$: Find nonzero $\vec{v}$ so that $A\vec{v} = -2\vec{v}$. i.e.,

$$A = \begin{pmatrix} -3 & 1 & -1 \\ -7 & 5 & -1 \\ -6 & 6 & -2 \end{pmatrix}$$

solve $(A + 2I)\vec{v} = \vec{0}$

$$\left(\begin{array}{ccc|c} -1 & 1 & -1 & 0 \\ -7 & 7 & -1 & 0 \\ -6 & 6 & 0 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} -1 & 1 & -1 & 0 \\ 0 & 0 & -1 & 0 \\ -1 & 1 & 0 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -1 & 1 & 0 & 0 \end{array}\right)$$

$\vec{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$ $\quad v_3 = 0$, $\quad v_1 = v_2$

$$= \begin{pmatrix} v_2 \\ v_2 \\ 0 \end{pmatrix} = v_2 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \quad v_2 \neq 0.$$

$\therefore$ For $\lambda = -2$, we get multiples of $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$.

$\therefore$ For $\lambda = 4$ we get nonzero multiples of $\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$.

Now what? Do we just get

$$u = c_1 e^{4t} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + c_2 e^{-2t} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} ?$$

as our general solution

NO

Note: $t = 0$ gives $u(0) = \underbrace{c_1 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}}$

Linear comb of only 2 vectors is a plane. Any initial data that is not on this plane is impossible to deal with.

$\therefore$ THIS doesn't work.

Apparently $u = c_1 e^{4t}\begin{pmatrix}0\\1\\1\end{pmatrix} + c_2 e^{-2t}\begin{pmatrix}1\\1\\0\end{pmatrix} +$ something else.

Recall: $\lambda = -2$ is repeated.

It's tempting to thing we need $\boxed{c_3 e^{-2t} t\vec{w}}$.

unfortunately, this won't work.

But, something of the form $c_3 \underbrace{e^{-2t}(t\vec{w} + \vec{v})}$ will work.

we need this to solve

$$\frac{d}{dt}\left(e^{-2t}(t\vec{w} + \vec{v})\right) = Ae^{-2t}(t\vec{w} + \vec{v})$$

$$(-2)\cancel{e^{-2t}}(t\vec{w} + \vec{v}) + \cancel{e^{-2t}}\vec{w} = \cancel{e^{-2t}}\left[tA\vec{w} + A\vec{v}\right]$$

$$-2\vec{w}t \quad -2\vec{v} + \vec{w} = t A\vec{w} \quad + A\vec{v}$$

(this has to be true for all t)

$$-2\vec{v} + \vec{w} = A\vec{v}$$

$$-2\vec{w} = A\vec{w}$$

$\therefore$ $\vec{w}$ is an eig. vector assoc. with $\lambda = -2$.

i.e. $\vec{w} = \alpha\begin{pmatrix}1\\1\\0\end{pmatrix}$

$$(A + 2I)\vec{v} = \alpha\begin{pmatrix}1\\1\\0\end{pmatrix}.$$

$$\begin{pmatrix} -1 & 1 & -1 \\ -7 & 7 & -1 \\ -6 & 6 & 0 \end{pmatrix}\vec{v} = \alpha\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$ Take $\alpha = 1$,

Solve $$\begin{pmatrix} -1 & 1 & -1 \\ -7 & 7 & -1 \\ -6 & 6 & 0 \end{pmatrix}\vec{v} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

<u>AND</u> I don't need every sol'n.

I need <u>a</u> sol'n.

$$\begin{pmatrix} \boxed{-1 \quad 1} & -1 & 1 \\ -7 & 7 & -1 & 1 \\ -6 & 6 & 0 & 0 \end{pmatrix} \xrightarrow{\frac{1}{6}R_3 \to R_3} \begin{pmatrix} \boxed{-1 \quad 1} & -1 & 1 \\ -7 & 7 & -1 & 1 \\ -1 & 1 & 0 & 0 \end{pmatrix}$$

$-R_3 + R_1 \to R_1$

$$\longrightarrow \begin{pmatrix} 0 & 0 & -1 & 1 \\ \boxed{-7 \quad 7} & \boxed{-1 \quad 1} \\ -1 & 1 & 0 & 0 \end{pmatrix}$$

$-R_1 + R_2 \to R_2$

$-7R_3 + R_2 \to R_2$

$$\begin{pmatrix} 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \end{pmatrix}$$

$v_3 = -1 \qquad v_1 = v_2$

$$\vec{v} = \begin{pmatrix} v_2 \\ v_2 \\ -1 \end{pmatrix}$$ where v_2 = anything.

Take $v_2 = 0$.

$$\vec{v} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}, \quad \vec{w} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

This gives $e^{-2t}\left(t\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}\right)$

Aside

$$\det\begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & -1 \end{pmatrix} = -1(-1) = 2 \neq 0$$

$$\Longrightarrow \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$$

are L.I.

as our other part. $\Rightarrow$ the general sol'n is

$$u = c_1 e^{4t}\begin{pmatrix}0\\1\\1\end{pmatrix} + c_2 e^{-2t}\begin{pmatrix}1\\1\\0\end{pmatrix} + c_3 e^{-2t}\left(t\begin{pmatrix}1\\1\\0\end{pmatrix} + \begin{pmatrix}0\\0\\-1\end{pmatrix}\right)$$

Note: $u(0) = c_1\begin{pmatrix}0\\1\\1\end{pmatrix} + c_2\begin{pmatrix}1\\1\\0\end{pmatrix} + c_3\begin{pmatrix}0\\0\\-1\end{pmatrix}$

Nonhomogeneous Problems

nonzero vector function.

$$u'(t) = Au(t) + f(t)$$

Solution Process:

homog. problem.

1. Get the general solution $u_h(t)$ to $u'(t) = A\,u(t)$.

2. Get any "particular solution" $u_p(t)$ to $u'(t) = A\,u(t) + f(t)$

3. The general solution is $u(t) = u_h(t) + u_p(t)$.

Example: Find the general solution to

$$\begin{cases} x' = x - 3y - e^t \\ y' = -2x + 2y + e^{2t} \end{cases} \qquad u = \begin{pmatrix} x \\ y \end{pmatrix}$$

(Use an analog of undetermined coefficients to find the particular solution.)

$$u' = \begin{pmatrix} 1 & -3 \\ -2 & 2 \end{pmatrix} u + \underbrace{\begin{pmatrix} -e^t \\ e^t \end{pmatrix}}_{f(t)}$$

1. Find the general sol'n to

$$u' = \begin{pmatrix} 1 & -3 \\ -2 & 2 \end{pmatrix} u$$

2. Find a particular sol'n to

$$u' = \begin{pmatrix} 1 & -3 \\ -2 & 2 \end{pmatrix} u + \begin{pmatrix} -e^t \\ e^t \end{pmatrix}$$

3. Add.

① Note: Eig. vals of $\begin{pmatrix} 1 & -3 \\ -2 & 2 \end{pmatrix}$ are -1 and 4.

$\lambda = -1$:

(1)A - (-1)I
2, -3;
-2, 3;

$\therefore$ $(A+I)\vec{v} = \vec{0}$ gives

$$2v_1 = 3v_2$$

$$\vec{v} = \begin{pmatrix} \frac{3}{2}v_2 \\ v_2 \end{pmatrix} = \frac{1}{2}v_2 \underline{\underline{\begin{pmatrix} 3 \\ 2 \end{pmatrix}}}$$

$\lambda = 4$: $(A-4I)\vec{v} = \vec{0}$ gives

$$\begin{pmatrix} -3 & -3 & 0 \\ -2 & -2 & 0 \end{pmatrix} \Rightarrow v_1 = -v_2$$

$$\vec{v} = \begin{pmatrix} -v_2 \\ v_2 \end{pmatrix} = v_2 \underline{\underline{\begin{pmatrix} -1 \\ 1 \end{pmatrix}}}$$

$$\Rightarrow u_h = c_1 e^{-t}\begin{pmatrix} 3 \\ 2 \end{pmatrix} + c_2 e^{4t}\begin{pmatrix} -1 \\ 1 \end{pmatrix}.$$

② Get u_p.

$$u_p' = \begin{pmatrix} 1 & -3 \\ -2 & 2 \end{pmatrix} u_p + \begin{pmatrix} -e^t \\ e^t \end{pmatrix}$$

i.e.

$$u_p' = \begin{pmatrix} 1 & -3 \\ -2 & 2 \end{pmatrix} u_p + \underbrace{\underline{\underline{e}}^t \begin{pmatrix} -1 \\ 1 \end{pmatrix}}_{\text{Not part of } u_h.}$$

Try $u_p = e^t \vec{w}$.

$$\Rightarrow u_p' = e^t \vec{w}$$

Substitute:

$$\underbrace{\cancel{e^t}\vec{w}}_{I\vec{w}} = \underline{\begin{pmatrix} 1 & -3 \\ -2 & 2 \end{pmatrix}}\cancel{e^t}\vec{w} + \cancel{e^t}\begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$I\vec{w} - \begin{pmatrix} 1 & -3 \\ -2 & 2 \end{pmatrix}\vec{w} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

Solve

$$\left(I - \begin{pmatrix} 1 & -3 \\ -2 & 2 \end{pmatrix}\right)\vec{w} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 3 \\ 2 & -1 \end{pmatrix}\vec{w} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 3 & -1 \\ 2 & -1 & 1 \end{pmatrix}$$

$3w_2 = -1$

Note:

$$\boxed{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 1 & -3 \\ -2 & 2 \end{pmatrix} = \begin{pmatrix} 0 & 3 \\ 2 & -1 \end{pmatrix}}$$

$$w_2 = -\frac{1}{3}$$

$$2w_1 + \frac{1}{3} = 1 \Rightarrow w_1 = \frac{1}{3}$$

$$\therefore \quad u_p = e^t\vec{w} = e^t\begin{pmatrix} 1/3 \\ -1/3 \end{pmatrix}$$

$$\therefore \quad u = \underbrace{c_1 e^{-t}\begin{pmatrix} 3 \\ 2 \end{pmatrix} + c_2 e^{4t}\begin{pmatrix} -1 \\ 1 \end{pmatrix}}_{u_h} + \underbrace{e^t\begin{pmatrix} 1/3 \\ -1/3 \end{pmatrix}}_{u_p}$$

you.

Example: Give the solution to

initial data.

$$u' = \begin{pmatrix} 3 & -1 \\ -1 & 3 \end{pmatrix} u + \begin{pmatrix} 4e^{2t} \\ 3e^{4t} \end{pmatrix},\ u(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Strategy: ① Get u_h solving

$$u_h' = \begin{pmatrix} 3 & -1 \\ -1 & 3 \end{pmatrix} u_h$$

② Get u_p solving

$$u_p' = \begin{pmatrix} 3 & -1 \\ -1 & 3 \end{pmatrix} u_p + e^{2t}\begin{pmatrix} 4 \\ 0 \end{pmatrix} + e^{4t}\begin{pmatrix} 0 \\ 3 \end{pmatrix}$$

Try $u_p = e^{2t}\vec{v} + e^{4t}\vec{w}$

unless 2 and/or 4 are eigenvectors.

They both are, so try

$$u_p = e^{2t}(t\vec{v} + \vec{z}) + e^{4t}(t\vec{w} + \vec{y})$$

③ $u = u_h + u_p$.

④ Satisfy $u(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

Example: Give the solution to

$$u' = \begin{pmatrix} 3 & -2 \\ 1 & 0 \end{pmatrix} u + \begin{pmatrix} -2e^{-t} \\ -2e^{-t} \end{pmatrix}, \ u(0) = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

Solutions to Selected Final Exam Review Problems

6.4 #8: Solve $x' = Ax$ where

$$A = \begin{pmatrix} -3 & 0 & -3 \\ 1 & -2 & 3 \\ 1 & 0 & 1 \end{pmatrix}.$$ Hint: -2 is an eigenvalue.

-3, 0, -3;
1, -2, 3;
1, 0, 1;
The eigenvalues are
-2 + 0 i
-2 + 0 i
0 + 0 i

$\lambda = 0$ and $\lambda = -2$

repeated.

$\lambda = -2$: Solve $(A - (-2)I)\vec{v} = \vec{0}$

$$\begin{pmatrix} -1 & 0 & -3 \\ 1 & 0 & 3 \\ 1 & 0 & 3 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$v_1 = -3v_3$ and $v_2 =$ anything

$$\Rightarrow \vec{v} = \begin{pmatrix} -3v_3 \\ v_2 \\ v_3 \end{pmatrix} = v_3 \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix} + v_2 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

L.I.

$\lambda = 0$: solve $(A - 0I)\vec{v} = \vec{0}$

$$\begin{pmatrix} -3 & 0 & -3 \\ 1 & -2 & 3 \\ 1 & 0 & 1 \end{pmatrix} \vec{v} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} -3 & 0 & -3 & 0 \\ 1 & -2 & 3 & 0 \\ 1 & 0 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 1 & -2 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \Rightarrow \cdots \ \vec{V} = \alpha \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}.$$

$$\therefore \quad X = c_1 e^{0t} \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} + c_2 e^{-2t} \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix} + c_3 e^{-2t} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

i.e. $$X = c_1 \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} + c_2 e^{-2t} \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix} + c_3 e^{-2t} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

Note: $$x(0) = c_1 \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} + c_2 \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix} + c_3 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

can be anything provided $\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}, \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ are L.I.

Check: $$\det \begin{pmatrix} 1 & -3 & 0 \\ -1 & 0 & 1 \\ -1 & 1 & 0 \end{pmatrix}$$

$$= -1 \begin{pmatrix} 1 & -3 \end{pmatrix} = 2 \neq 0.$$

OK.

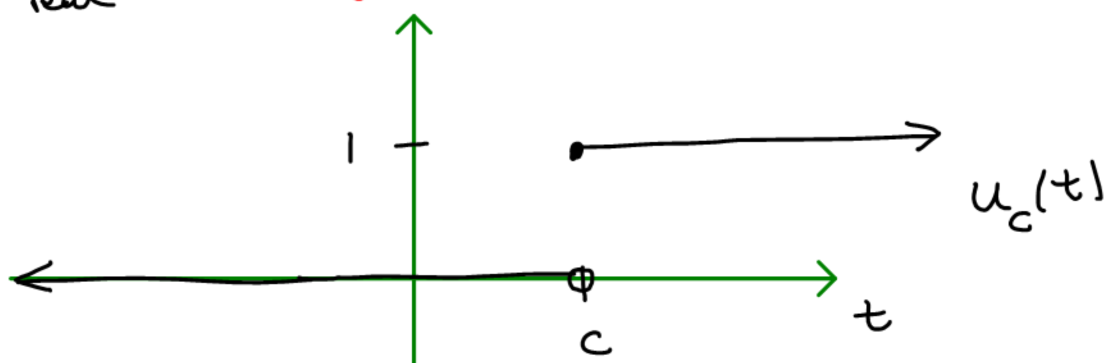
From the review:

9. Make sure you can use Heaviside functions to rewrite a piecewise defined function, and also make sure you can take the Laplace transform of a piecewise defined function.

Heaviside Functions:

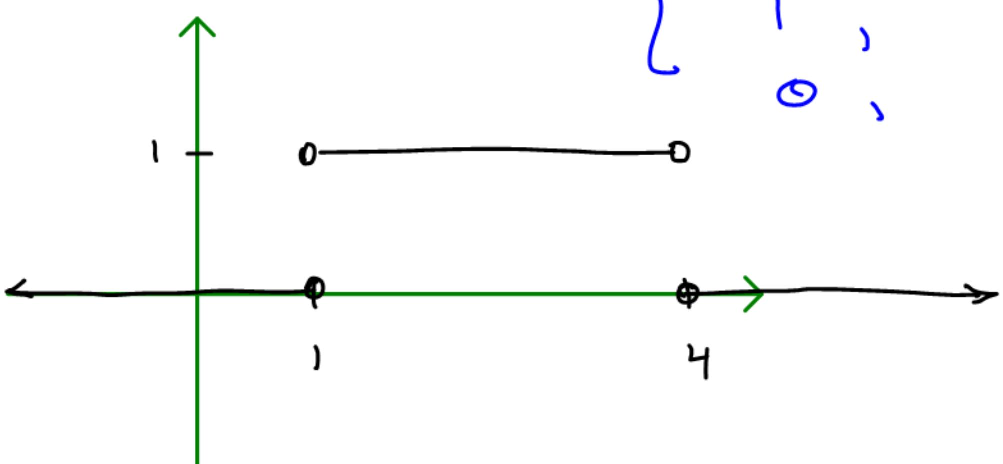
$$u_c(t) = \begin{cases} 0, & t < c \\ 1, & t \geq c. \end{cases}$$

c real #



Also called a unit step function.

ex: write $f(t) = \begin{cases} 0, & t < 1 \\ 1, & 1 < t < 4 \\ 0, & t > 4 \end{cases}$



Note: $f(t) = u_1(t) - u_4(t)$

except at $t = 1, t = 4$.

<u>Example</u>: Write $f(t) = \begin{cases} 0, & t < 0 \\ \cos(t), & 0 < t < 3 \\ \sin(t), & t > 3 \end{cases}$

using Heaviside functions.

$$f(t) = \cos(t)\big(u_0(t) - u_3(t)\big) + \sin(t)\,u_3(t)$$

$$= \cos(t)\,u_0(t) - \cos(t)\,u_3(t) + \sin(t)\,u_3(t)$$

<u>Note</u>: If we needed a Laplace transform, it would look like

$$\mathcal{L}\{f(t)\}(s) = \mathcal{L}\{\cos(t)u_0(t)\}(s) - \mathcal{L}\{\cos(t)\,u_3(t)\}(s) + \mathcal{L}\{\sin(t)\,u_3(t)\}.$$

$$= \mathcal{L}\{\cos(t)\}(s) - \mathcal{L}\{\cos(t)\,u_3(t)\}(s) + \mathcal{L}\{\sin(t)\,u_3(t)\}.$$

<u>Formula</u>: Spse $c > 0$.

$$\mathcal{L}\{g(t)\,u_c(t)\}(s) = \int_0^\infty e^{-st} g(t)\,\underline{\underline{u_c(t)}}\,dt$$

$$= \int_c^\infty e^{-st} g(t)\,dt$$

$t = c + z$

$z = t - c$

$$= \int_0^\infty e^{-s(c+z)} g(c+z)\,dz$$

$$\mathcal{L}\{g(t)u_c(t)\} = e^{-sc} \underbrace{\int_0^\infty e^{-sz} g(c+z)\,dz}$$

$$\mathcal{L}\{g(t)u_c(t)\} = e^{-sc}\mathcal{L}\{g(c+t)\}.$$

Ex: $\mathcal{L}\{e^{2t}u_3(t)\} = e^{-3s}\mathcal{L}\{e^{6+2t}\}$

$$\boxed{\begin{aligned} g(t) &= e^{2t} \\ g(3+t) &= e^{2(3+t)} \\ &= e^{6+2t} \end{aligned}}$$

$$= e^{-3s} \cdot e^{6} \cdot \mathcal{L}\{e^{2t}\}$$

$$= e^{6-3s} \cdot \frac{1}{s-2}.$$

Going backwards:

we know

$$\mathcal{L}\{g(t)u_c(t)\} = e^{-sc}\mathcal{L}\{g(c+t)\}.$$

$$\mathcal{L}\{h(t-c)u_c(t)\} = e^{-sc}\mathcal{L}\{h(t)\}$$

ex: Find a function whose Laplace Transform is

$$\underbrace{e^{-s}\frac{2}{s-1} - 3e^{-3s}\frac{s}{s^2+1}}$$

$$\left(e^{-s}\mathcal{L}\{2e^t\} - 3\underbrace{e^{-3s}\mathcal{L}\{\cos(t)\}}\right.$$

$$= \mathcal{L}\{2e^{t-1}u_1(t)\} - 3\mathcal{L}\{\cos(t-3)u_3(t)\}$$

$\therefore$ $2e^{t-1}u_1(t) - 3\cos(t-3)u_3(t)$ has

$$e^{-s}\cdot\frac{2}{s-1} - 3e^{-3s}\cdot\frac{s}{s^2+1}$$

as its Laplace Transform.

23. Solve $\begin{pmatrix} x' = -y + e^{-t} \\ y' = x + 1 \\ x(0) = -1,\ y(0) = 1 \end{pmatrix}$ $\longleftarrow$ see the discussion board.

5. The matrix A is 3 by 3, and $A^{-1} = \begin{pmatrix} -1 & 0 & -1 \\ 0 & 1 & -2 \\ 1 & 1 & 0 \end{pmatrix}$. Solve the system of equations given by $A\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$.

$$A\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$$

$$\Rightarrow \quad \underbrace{A^{-1}A}_{= I}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1}\begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$$

$$I\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1}\begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$$

$$\therefore \quad \begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1}\begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 & 0 & -1 \\ 0 & 1 & -2 \\ 1 & 1 & 0 \end{pmatrix}\begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} -1 \\ 5 \\ 0 \end{pmatrix}$$