Math 3321 Online
Review Problems

1. Solve $u'(t)+2u(t)=e^{-t}$, $u(0)=1$.

2. Give the general solution to $\dfrac{dy}{dx}=xy^2$.

3. Use elementary row operations to find the inverse of the matrix $\begin{pmatrix} -1 & 1 & -4 \\ -1 & 2 & -1 \\ -3 & 7 & 1 \end{pmatrix}$.

4. Use elementary row operations to solve the system $\begin{pmatrix} -x+4y-3z=1 \\ x-3y-z=2 \\ -x+3y+2z=-1 \end{pmatrix}$.

5. The matrix A is 3 by 3, and $A^{-1}=\begin{pmatrix} -1 & 0 & -1 \\ 0 & 1 & -2 \\ 1 & 1 & 0 \end{pmatrix}$. Solve the system of equations given by $A\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$.

6. Give the determinant of $\begin{pmatrix} -1 & 0 & 1 \\ 1 & -1 & -1 \\ -2 & 1 & 0 \end{pmatrix}$ by expanding across row 1.

7. Use Laplace transforms to find the solution to
   $u''(t)-u(t)=\exp(2t)$, $u(0)=1$, $u'(0)=0$.

8. Find the function whose Laplace transform is $\dfrac{1}{s-3}+\dfrac{1}{s^2+4}+\dfrac{3e^{-s}}{s}$.

9. Make sure you can use Heaviside functions to rewrite a piecewise defined function, and also make sure you can take the Laplace transform of a piecewise defined function.

10. The eigenvalues of $\begin{pmatrix} 7 & -4 \\ 12 & -7 \end{pmatrix}$ are 1 and -1. The eigenvectors associated with 1 are nonzero scalar multiples of $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and the eigenvectors associated with -1 are nonzero scalar multiples of $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$. Solve the initial value problem $\begin{pmatrix} x' = 7x - 4y \\ y' = 12x - 7y + e^{2t} \\ x(0) = -1,\ y(0) = 1 \end{pmatrix}$.

11. Determine the values of k for which the system $\begin{pmatrix} x + 2ky = 3 \\ kx + 3y = 1 \end{pmatrix}$ is inconsistent.

12. Give the form of a particular solution to $y''(t) - 3y'(t) - 4y(t) = \sin(t) + 2e^{-t}$.

13. Solve $u''(t) - 3u'(t) + 2u(t) = \exp(-t),\ u(0) = 1,\ u'(0) = 0$.

14. Use Euler's method with a step size of 0.1 to approximate $y(0.2)$ where y solves $y' = x + 2y,\ y(0) = 0$

15. Use Improved Euler's method with a step size of 0.1 to approximate $y(0.1)$ where y solves $y' = x + 2y,\ y(0) = 0$.

16. Suppose $A = \begin{pmatrix} 2 & 2 & 3 \\ 1 & 2 & 1 \\ 2 & -2 & 1 \end{pmatrix}$. Give the characteristic polynomial of A.

17. Suppose $A = \begin{pmatrix} 2 & 2 & 3 \\ 1 & 2 & 1 \\ 2 & -2 & 1 \end{pmatrix}$. Verify that the eigenvalues of A are -1, 4 and 2.

18. Suppose $A = \begin{pmatrix} 2 & 2 & 3 \\ 1 & 2 & 1 \\ 2 & -2 & 1 \end{pmatrix}$. Give all of the eigenvectors associated with the eigenvalue 4.

19. Give a value of c so that the system $\begin{pmatrix} -x+4y+z=1 \\ 2x-7y-2z=2 \\ x-3y-z=c \end{pmatrix}$ has at least one solution.

20. Show that the vectors $\left\{ \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 5 \\ -2 \end{pmatrix} \right\}$ are linearly dependent.

21. Explain why a linear system of 2 equations with 2 unknown either has 0, 1 or infinitely many solutions.

22. Solve $\begin{pmatrix} x' = 3x - y \\ y' = -x + 3y + e^{2t} \\ x(0) = -1,\ y(0) = 1 \end{pmatrix}$.

23. Solve $\begin{pmatrix} x' = -y + e^{-t} \\ y' = x + 1 \\ x(0) = -1,\ y(0) = 1 \end{pmatrix}$.

24. Rework all of the problems from the midterm exam, and rework all of the problems from the written homework and online quizzes.

25. Study the examples worked in the online sessions and posted in the online videos.

A Laplace transform formula sheet will be provided with the final exam.