Online Math 3321
Midterm – Review

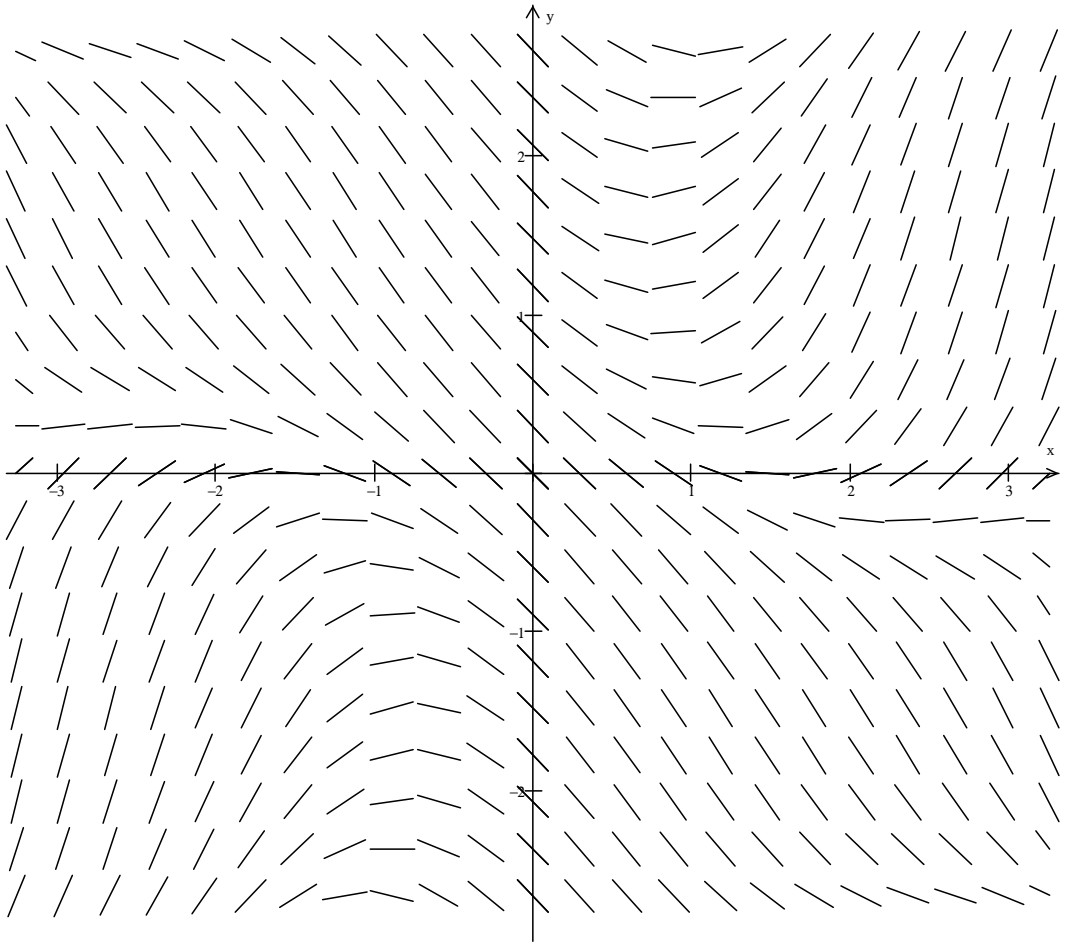
1. Give the solutions to each of the following:
   a. $y' = 2y, y(0) = 3$
   b. $y' + 3y = 0, y(0) = 2$
   c. $y' + y = 3, y(0) = -1$
   d. $y' = 3y, y(0) = 2$
   e. $y' + 2y = 0, y(0) = 3$
   f. $y' + 3y = 1, y(0) = 1$
2. Give the general solution to $y' + 3y = e^{-2t}$.

3. Give an integrating factor that can help solve the differential equation $y' + 3x\,y = \sin(x)$. Do not solve the differential equation.

4. A cup of coffee is found in a room at 8pm, and its temperature is determined to be 120°F at the time it is found. One hour later, the coffee's temperature is determined to be 90°F. There is a coffee maker in the room that brews coffee at 140°F. If the room is kept at a constant temperature of 68°F, then how long will it be from the time that the coffee was first found, until the coffee has a temperature of 75°F?

5. A 100 gallon tank is initially full of pure water. At time $t = 0$, a 30% hydrochloric acid solution begins to flow into the tank at the rate of 3 gallons per minute. The well-mixed solution is pumped out at the same rate. Find the time when the hydrochloric acid concentration becomes 20%.
6. Solve the initial value problem $y' = \dfrac{2x-3}{y^2}, y(0) = 1$.
7. Solve the initial value problem $y' = y^2(x+2), y(0) = 1$.
8. Solve the initial value problem $y' = \dfrac{xy + y^2}{2x^2}$, $y(1) = 2$.

9. Given the direction field below for the differential equation $y' = x\sin(y) - \cos(x)$, give the approximate sketch of the solution to $y' = x\sin(y) - \cos(x), y(-2) = -1$, and then give the approximate sketch of the solution to $y' = x\sin(y) - \cos(x), y(-2) = 2$.



10. Use Euler's method with a step size of 0.1 to approximate $u(0.2)$, where u solves $u' = -u + 2x,\ u(0) = 1$.
11. Use improved Euler's method with a step size of 0.1 to approximate $u(0.2)$, where u solves $u' = -u + 2x,\ u(0) = 1$.
12. Suppose the differential operator L is given by $L(u) = u'' - u' - 2u$. Find $L(\cos(x) + 2e^x)$.
13. Suppose the differential operator L is given by $L(u) = u'' + u' - 2u$. Find $L(\sin(3x) - e^{-2x})$.
14. Give the general solution to the homogeneous problem $y'' + 9y = 0$.
15. Give the general solution to the homogeneous problem $y'' - 2y' + y = 0$.
16. Give the general solution to the homogeneous problem $y^{(4)} - 3y'' + 2y = 0$.
17. Give the general solution to the homogeneous problem $y''' - 2y'' + 2y' = 0$.
18. Solve the initial value problem $y'' - 2y' + y = 0, y(0) = 2, y'(0) = 1$.

19. Solve the initial value problem $y''-2y'+y=-\sin(x)+e^{-x},\ y(0)=1,\ y'(0)=-1$.
20. Give the general solution to $y'''-2y''+2y'=3\sin(x)-1$.
21. Suppose you know $L\left[g'(x)\right]=\dfrac{e^{-2s}}{s^2+1}$ and $g(0)=2$. Determine $L\left[g(x)\right]$ WITHOUT finding a formula for $g(x)$.
22. Give the form of the Wronskian of any 2 solutions to the differential equation $y''+\cos(x)y'+xy=0$.
23. Give the Wronskian of $x\cos(x)$ and $\sin(x)$.
24. Give the general solution to $y''+y=\tan(x)$.
25. Give the Laplace transform of the solution to
$$y''-y'-2y=\cos(x)+e^{2x},\ y(0)=-1,\ y'(0)=2$$
without first solving the differential equation!!
26. Give the Laplace transform of the solution to
$$y''+y'-2y=\sin(x),\ y(0)=-1,\ y'(0)=2$$
without first solving the differential equation!!
27. Use Laplace transforms to solve the initial value problem $y''-y'-2y=e^{-x}+1,\ y(0)=1,\ y'(0)=-1$.
28. Use Laplace transforms to solve the initial value problem $y''-2y'-8y=2e^{3x},\ y(0)=1,\ y'(0)=-2$.
29. The differential equation $y''+2y'+y=0$ has linear independent solutions e^{-x} and xe^{-x}. Pretend you only know that xe^{-x} is a solution, and use reduction of order to obtain a second linearly independent solution.
30. **Use the definition of the Laplace transform to derive the formula** for the Laplace transform of $3e^{-2x}$.
31. **Use the definition of the Laplace transform to derive the formula** for the Laplace transform of $y'(x)$ in terms of the Laplace transform of $y(x)$.
32. **Use the definition of the Laplace transform formula** for the Laplace transform of $y'(x)$ in terms of the Laplace transform of $y(x)$, along with the table formula for the Laplace transform for $\cos(x)$ to derive the Laplace transform formula for $\sin(x)$.
33. Study all of the examples from class, the examples in the videos, the exercises in the homework, and the exercises in the online quizzes.