Online Math 3321
Midterm – Review

1. Give the solutions to each of the following:
   a. \( y' = 2y, \ y(0) = 3 \)
   b. \( y' + 3y = 0, \ y(0) = 2 \)
   c. \( y' + y = 3, \ y(0) = -1 \)
   d. \( y' = 3y, \ y(0) = 2 \)
   e. \( y' + 2y = 0, \ y(0) = 3 \)
   f. \( y' + 3y = 1, \ y(0) = 1 \)

2. Give the general solution to \( y' + 3y = e^{-2t} \).

3. Give an integrating factor that can help solve the differential equation \( y' + 3x \ y = \sin(x) \). Do not solve the differential equation.

4. A cup of coffee is found in a room at 8pm, and its temperature is determined to be 120°F at the time it is found. One hour later, the coffee’s temperature is determined to be 90°F. There is a coffee maker in the room that brews coffee at 140°F. If the room is kept at a constant temperature of 68°F, then how long will it be from the time that the coffee was first found, until the coffee has a temperature of 75°F?

5. A 100 gallon tank is initially full of pure water. At time \( t = 0 \), a 30% hydrochloric acid solution begins to flow into the tank at the rate of 3 gallons per minute. The well-mixed solution is pumped out at the same rate. Find the time when the hydrochloric acid concentration becomes 20%.

6. Solve the initial value problem \( y' = \frac{2x - 3}{y^2}, \ y(0) = 1 \).

7. Solve the initial value problem \( y' = y^2 (x + 2), \ y(0) = 1 \).

8. Solve the initial value problem \( y' = \frac{xy + y^2}{2x^2}, \ y(1) = 2 \).
9. Given the direction field below for the differential equation \( y' = x \sin(y) - \cos(x) \), give the approximate sketch of the solution to \( y' = x \sin(y) - \cos(x) \), \( y(-2) = -1 \), and then give the approximate sketch of the solution to \( y' = x \sin(y) - \cos(x) \), \( y(-2) = 2 \).

10. Use Euler’s method with a step size of 0.1 to approximate \( u(0.2) \), where \( u \) solves \( u' = -u + 2x \), \( u(0) = 1 \).

11. Use improved Euler’s method with a step size of 0.1 to approximate \( u(0.2) \), where \( u \) solves \( u' = -u + 2x \), \( u(0) = 1 \).

12. Suppose the differential operator \( L \) is given by \( L(u) = u'' - u' - 2u \). Find \( L(\cos(x) + 2e^x) \).

13. Suppose the differential operator \( L \) is given by \( L(u) = u'' + u' - 2u \). Find \( L(\sin(3x) - e^{-2x}) \).

14. Give the general solution to the homogeneous problem \( y'' + 9y = 0 \).

15. Give the general solution to the homogeneous problem \( y'' - 2y' + y = 0 \).

16. Give the general solution to the homogeneous problem \( y^{(4)} - 3y'' + 2y = 0 \).

17. Give the general solution to the homogeneous problem \( y''' - 2y'' + 2y' = 0 \).

18. Solve the initial value problem \( y'' - 2y' + y = 0 \), \( y(0) = 2 \), \( y'(0) = 1 \).
19. Solve the initial value problem $y'' - 2y' + y = -\sin(x) + e^{-x}$, $y(0) = 1$, $y'(0) = -1$.

20. Give the general solution to $y''' - 2y'' + 2y' = 3\sin(x) - 1$.

21. Suppose you know $L[g'(x)] = \frac{e^{-2x}}{s^2 + 1}$ and $g(0) = 2$. Determine $L[g(x)]$ WITHOUT finding a formula for $g(x)$.

22. Give the form of the Wronskian of any 2 solutions to the differential equation $y'' + \cos(x)y' + xy = 0$.

23. Give the Wronskian of $x\cos(x)$ and $\sin(x)$.

24. Give the general solution to $y'' + y = \tan(x)$.

25. Give the Laplace transform of the solution to $y'' - y' - 2y = \cos(x) + e^{2x}$, $y(0) = -1$, $y'(0) = 2$ without first solving the differential equation!!

26. Give the Laplace transform of the solution to $y'' + y' - 2y = \sin(x)$, $y(0) = -1$, $y'(0) = 2$ without first solving the differential equation!!

27. Use Laplace transforms to solve the initial value problem $y'' - y' - 2y = e^{-x} + 1$, $y(0) = 1$, $y'(0) = -1$.

28. Use Laplace transforms to solve the initial value problem $y'' - 2y' - 8y = 2e^{3x}$, $y(0) = 1$, $y'(0) = -2$.

29. The differential equation $y'' + 2y' + y = 0$ has linear independent solutions $e^{-x}$ and $xe^{-x}$.

   Pretend you only know that $xe^{-x}$ is a solution, and use reduction of order to obtain a second linearly independent solution.

30. Use the definition of the Laplace transform to derive the formula for the Laplace transform of $3e^{-2x}$.

31. Use the definition of the Laplace transform to derive the formula for the Laplace transform of $y'(x)$ in terms of the Laplace transform of $y(x)$.

32. Use the definition of the Laplace transform formula for the Laplace transform of $y'(x)$ in terms of the Laplace transform of $y(x)$, along with the table formula for the Laplace transform for $\cos(x)$ to derive the Laplace transform formula for $\sin(x)$.

33. Study all of the examples from class, the examples in the videos, the exercises in the homework, and the exercises in the online quizzes.