Vertical Teams

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Getting Started…

- Introductions
- Looking Ahead (a Snapshot of College Mathematics)
- Challenges – Something to Sleep On
- The Importance of Vertical Teams
- In and Out Activity
- Back Mapping – Using an AP Question to look at adaptions for levels grade 6- 12
- Streaks Example – Discovering Pattern in Random Processes – Utilizing the calculator in a nontraditional way.
- Area Approximation
- A Crazy Word Problem
- A Maximization Problem and an Associated Probability Question
- A Simple Formula for Areas of Polygons

Looking Ahead

Your Students…
Where Are They Headed?
What Do They Need?

Shameless Advertising

- High School Mathematics Contest
  http://mathcontest.uh.edu
- Houston Area Calculus Teachers Association AP Calculus Workshops
  http://www.HoustonACT.org
- EatMath Algebra I Teacher Workshops
  http://www.EatMath.org

(continued)

- Online Practice AP Testing in Calculus and Statistics
  (info at http://www.HoustonACT.org)
- Online Practice Algebra II EOC Testing (Fall 2011) – email bekki@math.uh.edu
- Online Help Materials
  (see http://online.math.uh.edu)
- *teach*HOUSTON

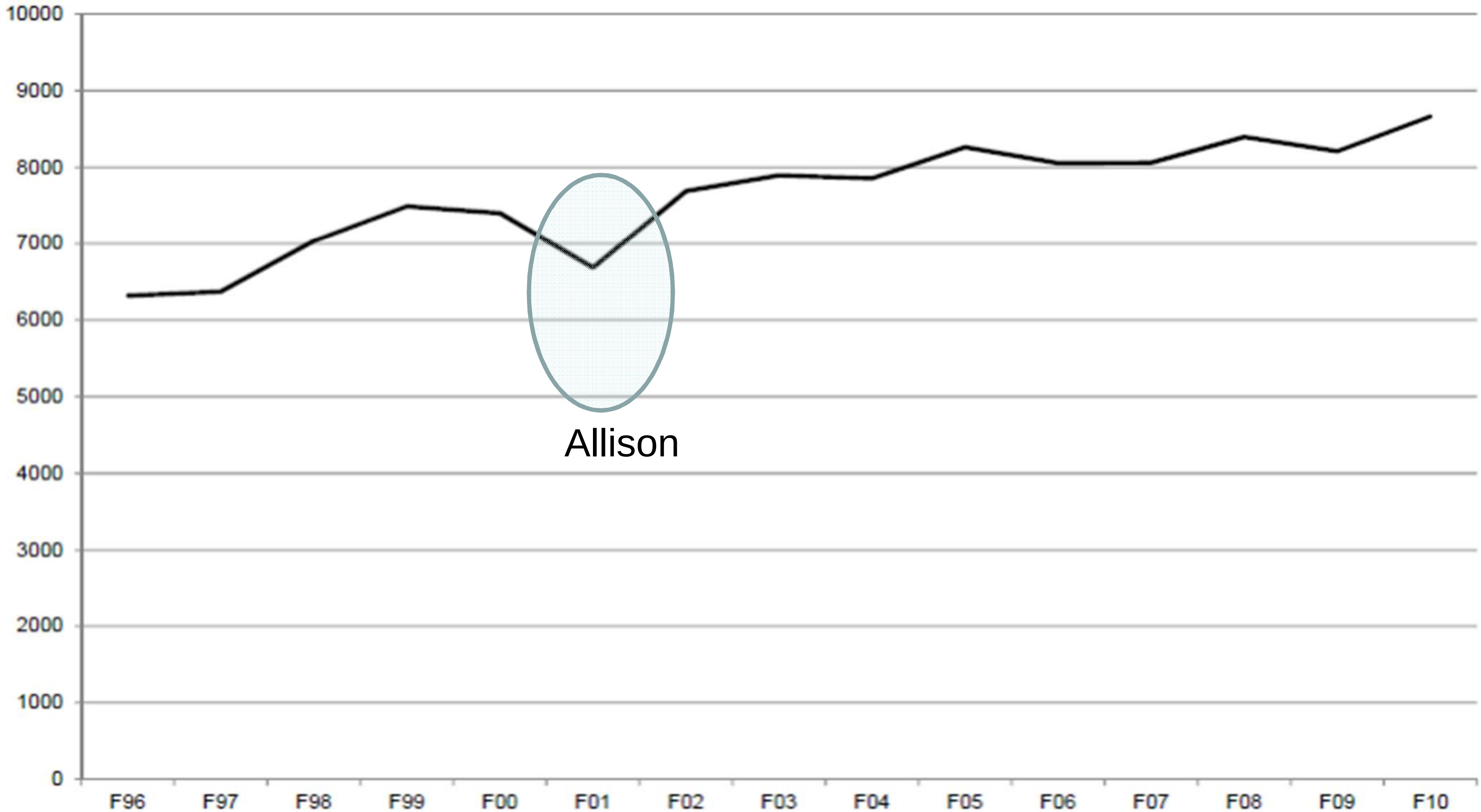
Technology Tool Tips

- PDF Annotator
- Mimio Notebook
- Geogebra
- Google SketchUp
- WinPlot
- Bamboo Tablet
- Graph
- Excel

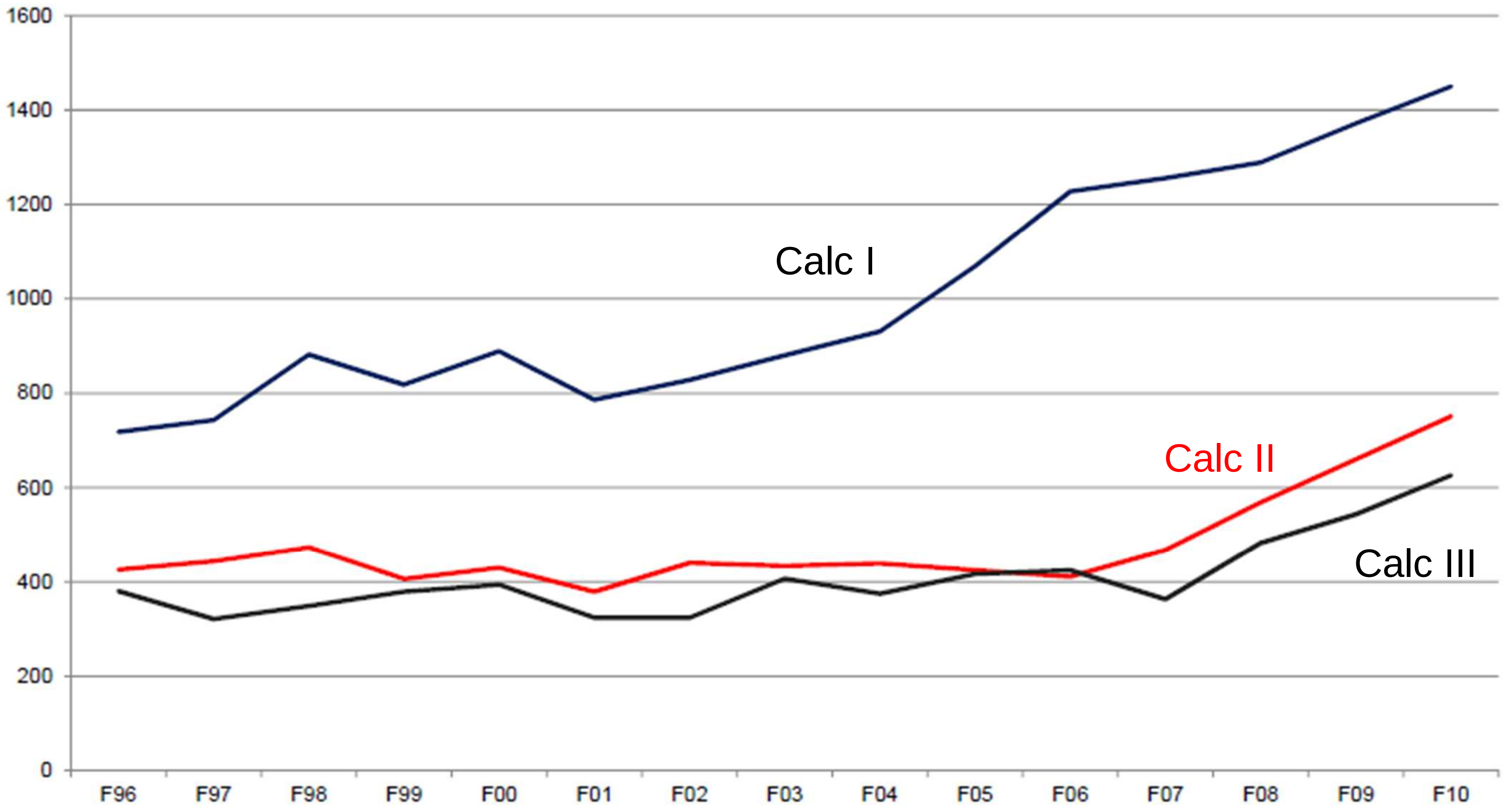
Freshmen Level Mathematics at UH

(a snapshot)

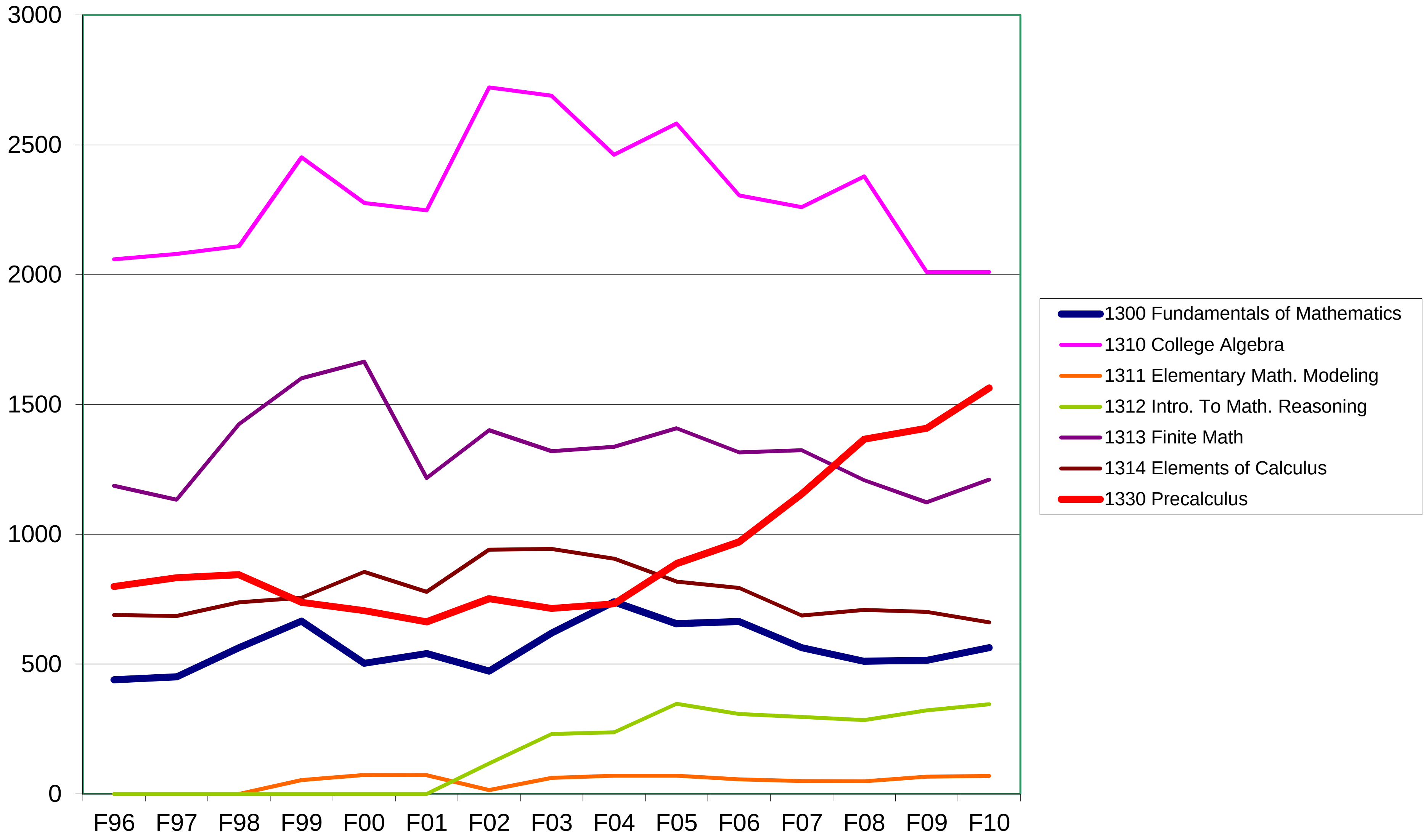
Freshmen Math Enrollment



Calculus I, II, III Enrollment



Enrollment Below Calculus



Faculty Concerns

- Prerequisite Knowledge
- Student Performance

Previous Faculty Attitude

Essentially no course coordination, and very few grades.

They are adults.
Let them find their own way.

Current Attitude

Total course coordination, and many grades.

They are young. Encourage them to work hard and be responsible.

The Bottom Line

Students will only work as hard as you make them work.

If you want more from your students, then demand more.

Actions In Freshmen Mathematics

(since 2003)

- Improved Placement Testing - Fall 2007
- No Adjunct and Essentially No Graduate Student Instruction
- Mandatory Attendance
- Daily Class Grades
- Daily Homework
- Weekly Online Quizzes
- Online Course Materials
- Increased Tutoring Availability
- Common Exams and Common Grading
- **No Curves!!**
- Improved Instruction and Course Coordination
- Faculty Peer Pressure

Performance Data - College Algebra

(College Algebra = Algebra II)

1310 Fall	A	B	C	D	F+W
%	32	20	20	12	16
Cum. %	32	52	72	84	100

According Data - Responsible students succeed in College Algebra.

Performance Data - PreCalculus

1330 Fall	A	B	C	D	F+W
%	24	18	15	12	31
Cum. %	24	42	57	69	100

This course is traditionally hard at every university.

Performance Data - Calculus I

1431 Fall	A	B	C	D	F+W
%	21	23	21	13	22
Cum. %	21	44	65	78	100

Which *Missing* Skills Are Crippling Students in College?

(…the same ones that cripple them in your classroom…)

- Arithmetic
- Simple Algebraic Manipulation
- Graphing and Understanding Basic Shapes
- Simple Geometry
- Critical Thinking
- Work Ethic
- Responsibility

A Nontrivial Number of Students Blindly Accept the Following

$$|a+b| = |a| + |b|$$

$$\sqrt{a^2 + b^2} = a + b$$

$$\sqrt{(x+1)^2} = x + 1$$

In General…

- Students do not know how to experiment.
- Students have seen so many mathematical topics that they do not know any of them very well.
- Students struggle to organize their work and explain what they have done.
- Students have learned processes that they do not understand.

…and it’s not your fault…

Important Geometric Concepts

- Pythagorean Theorem
- Areas of triangles, rectangles and circles.
- Circumference of rectangles and circles.
- The sum of the angles in a triangle.
- Similar triangles.
- Isosceles and equilateral triangles.
- Triangle trigonometry (if they are proceeding to calculus).

Honestly, that's it!

In General, We Are Happy If Students

- Can do simple arithmetic and basic algebra.
- Know everything about lines and parabolas.
- Can solve linear and quadratic equations, and linear inequalities.
- Can factor simple quadratics, complete the square and use the quadratic formula.
- Understand asymptotes (for calculus).
- Understand functions.
- Know the shapes of basic functions.
- Know the area formulas for circles, rectangles and triangles, and know the perimeter of a circle and a rectangle.
- Know basic trigonometry and the values of the trig functions at the special angles (for calculus).
- Can organize their work.
- Can act responsibly and are willing to work hard.

Of Course…

We would be thrilled if students could read and write mathematics!

Discussion Point: Why is this so difficult?

Geometry Challenge

Something to Sleep On

Is it possible to cut a circular disk into 2 or more congruent pieces so that at least one of the pieces does not “touch” the center of the disk?

Probability Challenge

Something to Sleep On

Pick a value in the first row. Then move forward that number from left to right and top to bottom. Keep going until you cannot complete a process.

4	1	5	3	3	5	2	4
3	2	2	5	1	5	2	5
2	4	2	1	3	4	2	3
3	5	4	3	2	3	3	3
1	1	1	3	5	5	5	5
1	2	1	5	5	5	3	3

In this case, you will always land on the 4^{th} entry in the last row. Something similar will happen nearly EVERY time a list of numbers is generated in a random manner.

Question: Can you create a grid where the last value can be different depending upon where you start?

Vertical Exploration and Discussion

http://apcentral.collegeboard.com/apc/members/repository/ap03_adaptation_calca_29895.pdf

The Importance of Vertical Teams

Dixie Ross

There is probably nothing more obvious and important in preparing kids to be successful in AP classes than the concept of vertical teaming. Math in particular seems to lend itself well to this process because it is so sequential in nature and since new learning is dependent on mastery of previously taught skills and concepts. When I first became interested in teaching AP calculus in a small, rural school in Texas over twenty years ago, I certainly did not know what vertical teaming was, but I knew that we couldn't just throw students as seniors into an AP Calculus class. While most teachers and administrators thought I was delusional to think that our students could be successful in the AP program, there was one other person who shared my vision. Her name is Carol Lindell and she was the department chair at the middle school that fed into my high school. She agreed that our students were just as good as students anywhere else and said she would make sure that they had a strong background from the middle school while I would pick them up as freshmen in geometry and carry them through their next few years to see that they were well prepared for calculus by the end of their junior year.

We collaborated regularly about what was important for the kids to know and what could be de-emphasized or eliminated from our overstuffed curriculum. Most importantly, we worked together to instill a change in attitude amongst parents, school personnel and the students themselves about what they were capable of achieving. I often brought the high school students over to the middle school to serve as inspiration and examples to the younger kids about what was possible when you worked hard and kept your eyes on a particular goal.

As expected, our first group of AP calculus exam takers in 1990 did quite well and we were convinced that our approach had merit. A few years later, we were asked to make a presentation at a workshop on vertical teaming and we asked "what is that?" and realized that what we had been doing all along actually had a name.

In 1994, I served on the committee that produced the Mathematics Vertical Teams Toolkit for the College Board. Working with administrators and teachers from many different types of schools across the nation, I began to realize how difficult vertical teaming can be in situations where you have multiple high schools within a district being fed by several different middle schools. Nonetheless, people from all over in many different situations were finding vertical teaming to be a valuable tool in building an inclusive AP program that served students with a variety of backgrounds. There was widespread recognition that bringing groups of teachers together and giving them time to discuss issues related to alignment of curriculum, sharing of best instructional practices and proper methods of assessing students deep understanding of challenging material was a smart use of resources.

When AP Central got up and running in 2002, it seemed like the perfect vehicle through which to share what we had learned from our own efforts in vertical teaming in math. There are several article posted there ("A Year in the Life of an AP Vertical Team for Mathematics" and "Adapting AP Questions as a Pre-AP Strategy") that can give teams some ideas of what we have done and what they might do to get their process started.

In and Out

Brainstorm the skills and concepts that you find would be beneficial for students to have mastered when entering your course (agree on 3-4) and then those skills and concepts you believe students will have mastered when leaving your course/level.

The Goals of the Pre-AP* Mathematics Program

Pre-AP* mathematics classes should do the following:

- Prepare growing numbers of students, especially those traditionally underrepresented in AP* courses, for the challenges offered by the Advanced Placement Program*
- Introduce skills, concepts and assessment methods to prepare students for success in AP and other challenging courses
- Require students to work with functions represented in a variety of ways (graphical, numerical, analytical, or verbal) and understand the connections among these representations
- Encourage students to develop their communication skills in mathematics to be able to read and interpret problem situations and explain solutions to problems both orally and in well-written sentences
- Provide students with multiple opportunities to model a written description of a physical situation with a function or to determine appropriate functions to match data sets that they are given or that they develop through experimentation or research
- Make regular use of technology to help solve problems, experiment, interpret results, and verify conclusions

- Require students to determine the reasonableness of their solutions, including sign, size, relative accuracy, and units of measurement
- Help students to develop an appreciation of mathematics as a coherent body of knowledge and as a human accomplishment
- Be a part of a well-planned and coherent curriculum so that teachers can build upon knowledge and skills that students have acquired in previous courses and can prepare students for subsequent courses
- Allow students to develop the work ethic and habits of mind that are necessary for success in the Advanced Placement Program and in other challenging mathematics programs

Many of these goals have been adapted from The Teacher's Guide to AP Calculus *and the Pre-AP brochure from the College Board.*

http://www.tealighthouse.org/math/

What Makes a Good Pre-AP* Mathematics Problem?

After reading the goals of the Pre-AP* mathematics program, seeing how AP* problems can be adapted for use at other levels. After examining exemplar problems for particular TEKS, we hope you have a better understanding of what makes a problem or activity particularly appropriate for use in the Pre-AP mathematics classroom. Below you will find some of the criteria that the committee used in selecting the problems and activities for this document.

A good Pre-AP mathematics problem or activity

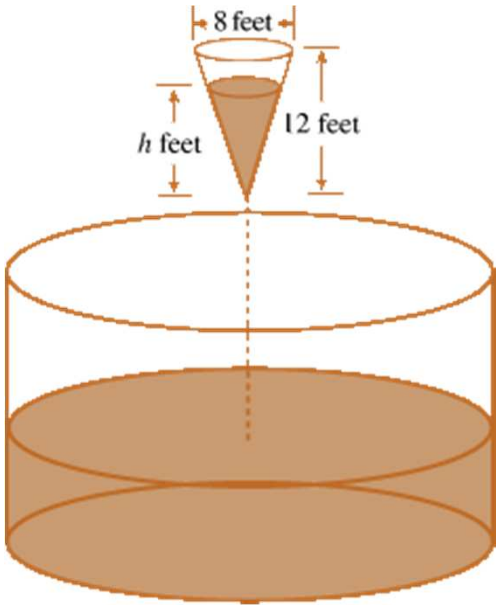
- has a clear connection to the vocabulary, skills, concepts, or habits of mind necessary for success in AP mathematics courses;
- goes beyond a minimalist approach to addressing the TEKS;
- can serve multiple purposes, such as addressing an Algebra I TEKS, reviewing a middle school geometry skill, and introducing an AP Calculus concept;
- should go beyond simple drill and recall (There should be a greater emphasis on analysis, application, and synthesis of material.);
- requires students to engage in an extended chain of reasoning (Problems should require more than one step and might cover more than one topic.);
- might be completely different from problems that the teacher has demonstrated in class, though based on the same concept (Students are expected to apply their knowledge in novel situations with very little teacher direction.);

- requires students to develop their reading and interpretation skills using verbal, graphical, analytical, and numerical prompts;
- asks students to communicate their thoughts orally and/or in writing (Students must be able to justify their work in clear, concise, and well-written sentences.);
- stretches the students in ways that might make them uncomfortable (The solving of problems might take several attempts. They might have to hear someone else's explanation (preferably one of their peers) before they begin to develop understanding.);
- should be graded based on the process and methods as well as the final answers; and
- might require the thoughtful use of technology.

http://www.tealighthouse.org/math/goodprob.php

Back-Mapping
AP* Calculus AB Problem 5, 1995

This problem was one of the six free-response questions on the 1995 AP* Calculus AB exam. Students had a great deal of difficulty in recognizing the similarity of the water in the cone to the cone figure itself. In Part A, they seemed to have difficulty with the wording of the problem. Had it said, "Solve for V in terms of h," a greater number of students probably would have solved the equation correctly. Students also had difficulty in working with the volume formulas and with the simple symbolic manipulation that was required. All of these difficulties occurred well before the students had a chance to reach the calculus stage of the problem. The concepts of proportionality, similarity, volume, and symbolic manipulation are introduced as early as middle school, but perhaps could be approached a little differently in a Pre-AP* situation. In addition to examining the similarity of plane figures, for example, Pre-AP students could also examine the similarity of three-dimensional figures and embedded figures. Instead of just using formulas to calculate area or volume, there could be more emphasis on manipulating the formulas and expressing them in different forms. The same TEKS would be addressed but in a way that will better prepare students for the AP experience.



5. As shown in the figure above, water is draining from a conical tank with height 12 feet and diameter 8 feet into a cylindrical tank that has a base with area 400π square feet. The depth h, in feet, of the water in the conical tank is changing at the rate of (h-12) feet per minute. (The volume V of a cone with radius r and height h is $V = 1/3\pi r^2 h$.)

(a) Write an expression for the volume of water in the conical tank as a function of h.

(b) At what rate is the volume of water in the conical tank changing when h = 3? Indicate units of measure.

(c) Let y be the depth, in feet, of the water in the cylindrical tank. At what rate is y changing when h = 3? Indicate units of measure.

Discussion of Student Worksheets

- Middle School Activity 2
- Middle School Activity 3
- Geometry Activity 1
- Geometry Activity 2
- Vertical Team Discussion

Your Turn…
Vertical Team Problem Based on AP Calculus 2005 AB4

Let f be a function that is continuous on the interval [0,4). The function has the properties indicated in the table below. Your **first task** is to sketch the graph of a function that has all the characteristics of f. Then answer the questions that follow.

x	**0**	**$0<x<1$**	**1**	**$1<x<2$**	**2**	**$2<x<3$**	**3**	**$3<x<4$**
f(x)	-1	negative	0	positive	2	positive	0	**negative**
direction		increasing		increasing		decreasing		**decreasing**
curvature		**concave down**		**concave up**		**concave down**		**concave up**

Follow-up Questions

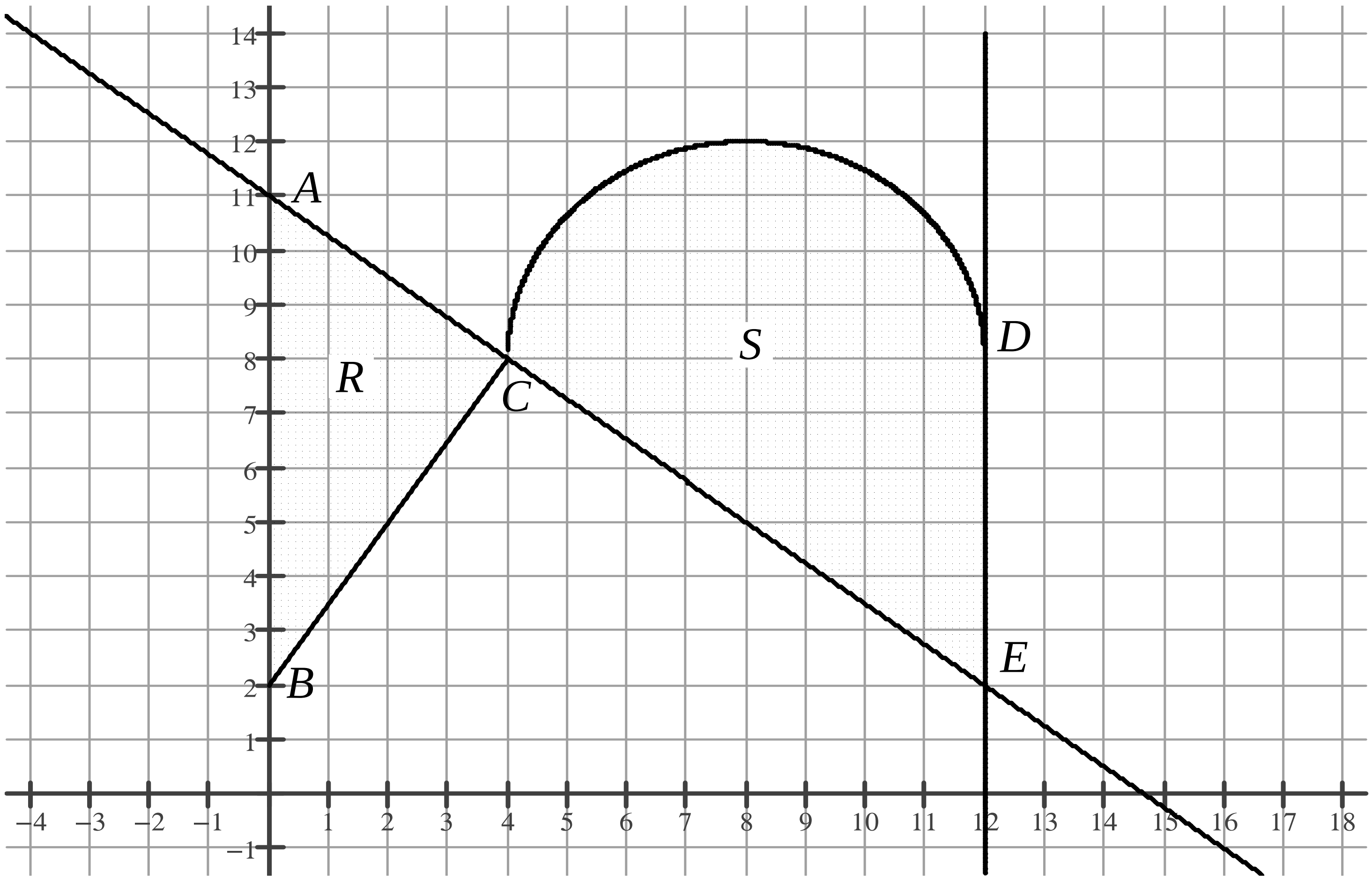
1. What is the range of the function you have graphed?
2. What is the minimum value of the function?
3. Where does the maximum value of the function occur?
4. How do you know from the given information that this graph does NOT have a vertical asymptote?

Second Task

On one of the index cards you have been given, you will need to sketch a function. On the second index card, you will describe your function and write 4 follow-up questions.

You will have seventeen minutes to complete this task. When time is up, you will pass your description card on to another group along with the third index card so that they can attempt to draw your function.

Vertical Team Problem based on AP Calculus 2005 AB 1



Let R be the shaded region in the first quadrant enclosed by the y-axis and the two linear functions shown above. Let S be the shaded region in the first quadrant enclosed by the graphs of the linear function, the vertical line and the semi-circle centered at (8,8).

1. Find the area of region S.
2. Determine the equations for lines AE and BC.
3. Find the perimeter of region R.
4. Give the coordinates of the points A, B, C, D, and E.
5. What is the length of segment AB?
6. Solve (algebraically) the system of equations for lines AE and BC.
7. Draw a line segment perpendicular to the y-axis to point C. What is the length of this segment?
8. Describe the solid formed when region R is revolved around the y-axis.
9. Find the perimeter of region S.
10. Find the area of region R.
11. Find the volume of the solid formed by revolving region R around the y-axis.
12. Think of at least two more questions.

What Does Random Look Like?

Suppose you perform the following experiment:

Flip a fair coin 100 times.

What do you expect to see?

Activity: Send one student out of the room. Then divide the remaining students into groups A and B. Record the names of the students in each group, and give the students the following task.

Group A: Each student will flip a fair coin 100 times, recording the results in the order they occur by writing an H for "heads" and a T for "tails."

Group B: Each student will simulate the task of flipping a coin 100 times by writing H for "heads" and T for "tails" on a sheet of paper in the manner they think the flips will occur.

Now invite the student back into the room. Have them view the student H/T lists and guess their group.

Exploring with TI Basic

We can use the TI calculator to simulate 100 experiments of flipping a coin 100 times, where we keep track of the maximum streak length in each experiment.

The program on the right uses J to keep track of the experiment. In each experiment, a list of length 100 containing random 0's and 1's is stored in L1. Then L2 is created to keep track of the streak lengths. Finally, the maximum value of the streaks for the experiment is recorded in L3(J). We display J at each step to keep track of the progress.

For(J,1,100)

randInt(0,1,100) → L_1

1 → L_2(1)

For(K,2,100)

If L_1(K) = L_1(K-1)

Then

L_2(K-1)+1 → L_2(K)

Else

1 → L_2(K)

End

End

max(L_2) → L_3(J)

Disp J

End

What Does Random Look Like

(Part II)

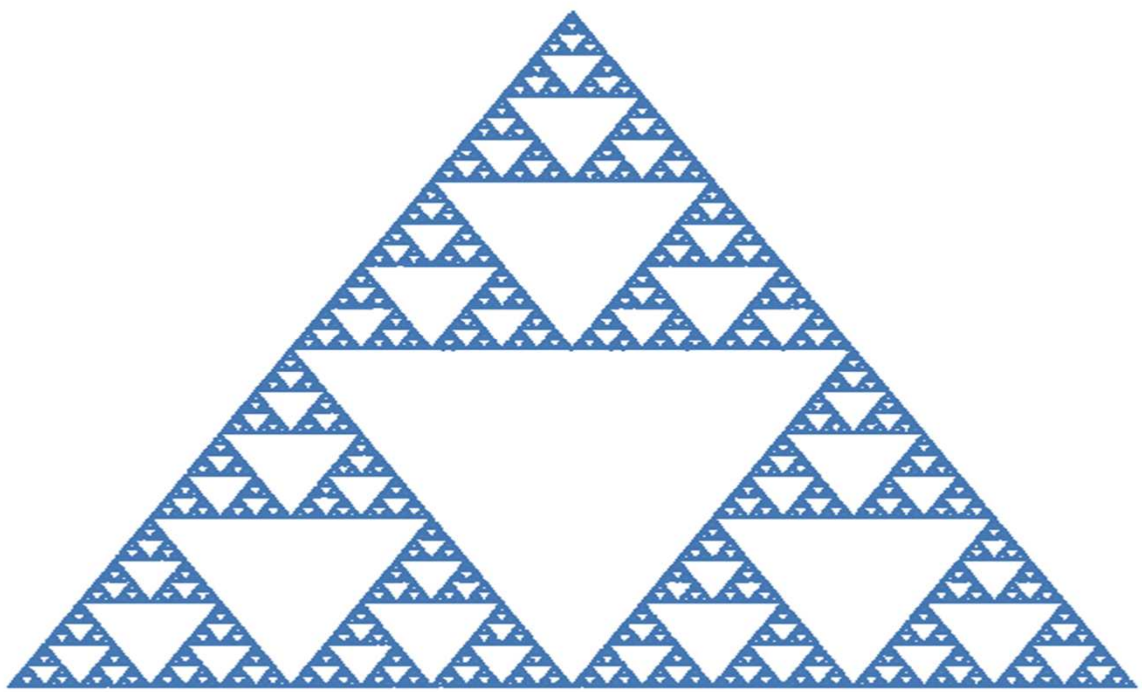
Create 3 points in the xy plane and label them A, B and C. Then pick a point P in the plane (at random) and plot it. Now pick a random integer from the set {1,2,3}. If the number selected is 1, then plot the midpoint between P and A. If the number selected is 2, then plot the midpoint between P and B. Otherwise, plot the midpoint between P and C. Whichever point you plot, call this new point P (removing the name from the old point P), and repeat the process MANY times (e.g. thousands of times).

Question: Will any pattern appear, or will the resulting sketch look like a complete mess?

What Does Random Look Like

(Part II)

Amazing Answer: Regardless of the starting point, if you throw out the first 20 points, the remaining points will given a plot which looks like the one below.



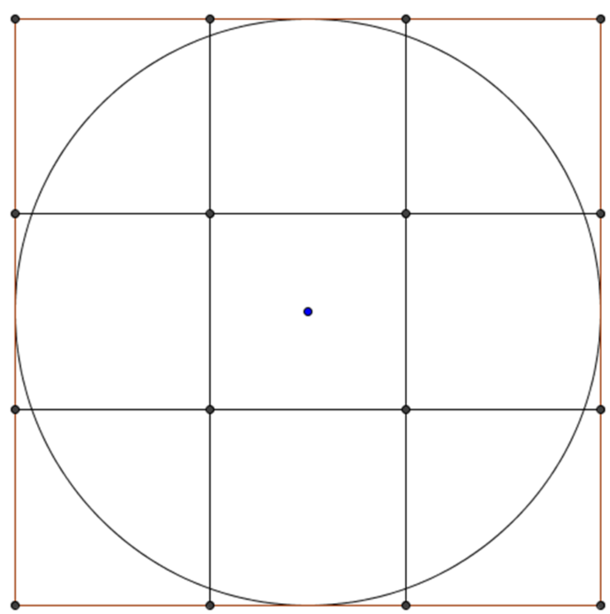
Note: This figure is called the Serpinski triangle.

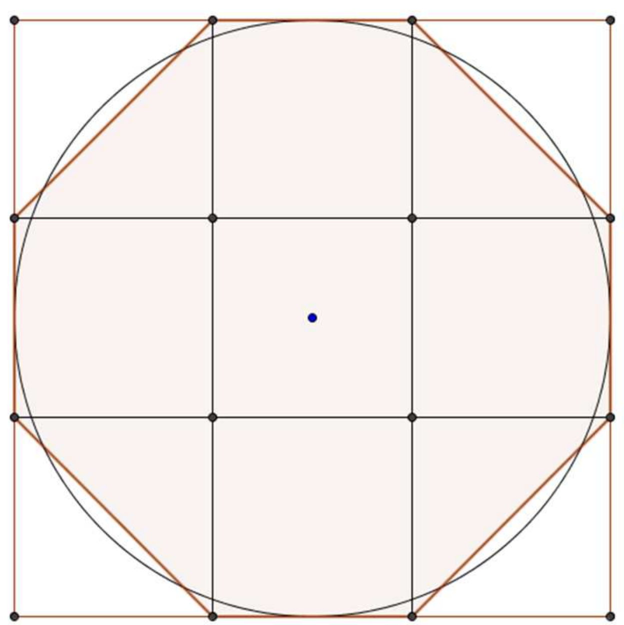
A 10 Minute Quick Look at Geogebra

Additional video introductions can be found at http://www.math.uh.edu/~jmorgan/Rice

Approximating Area – Part I

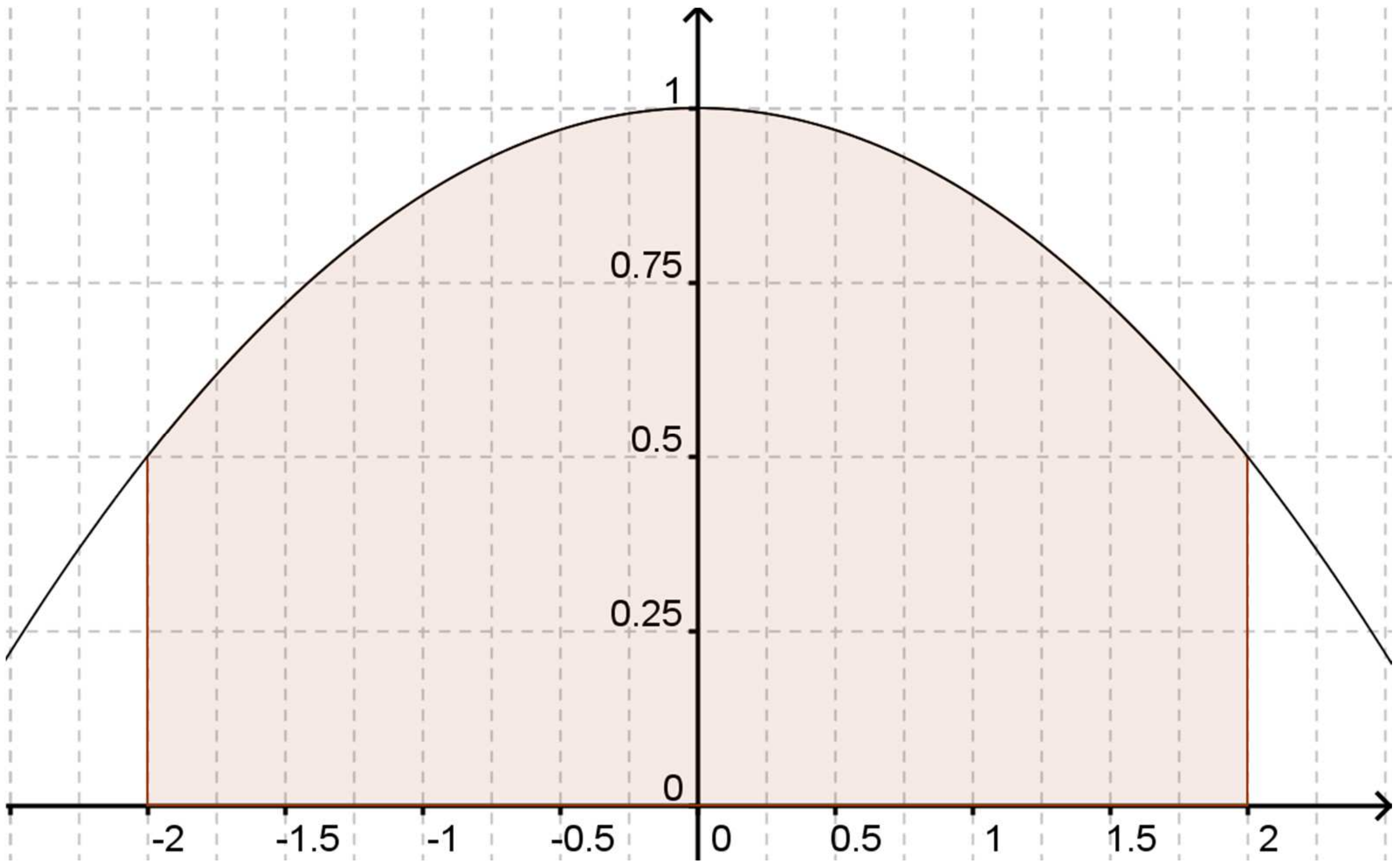
The ancient Egyptians used basic shapes to approximate the area of a circle. What value do you think they used to approximate π?





Approximating Area – Part II

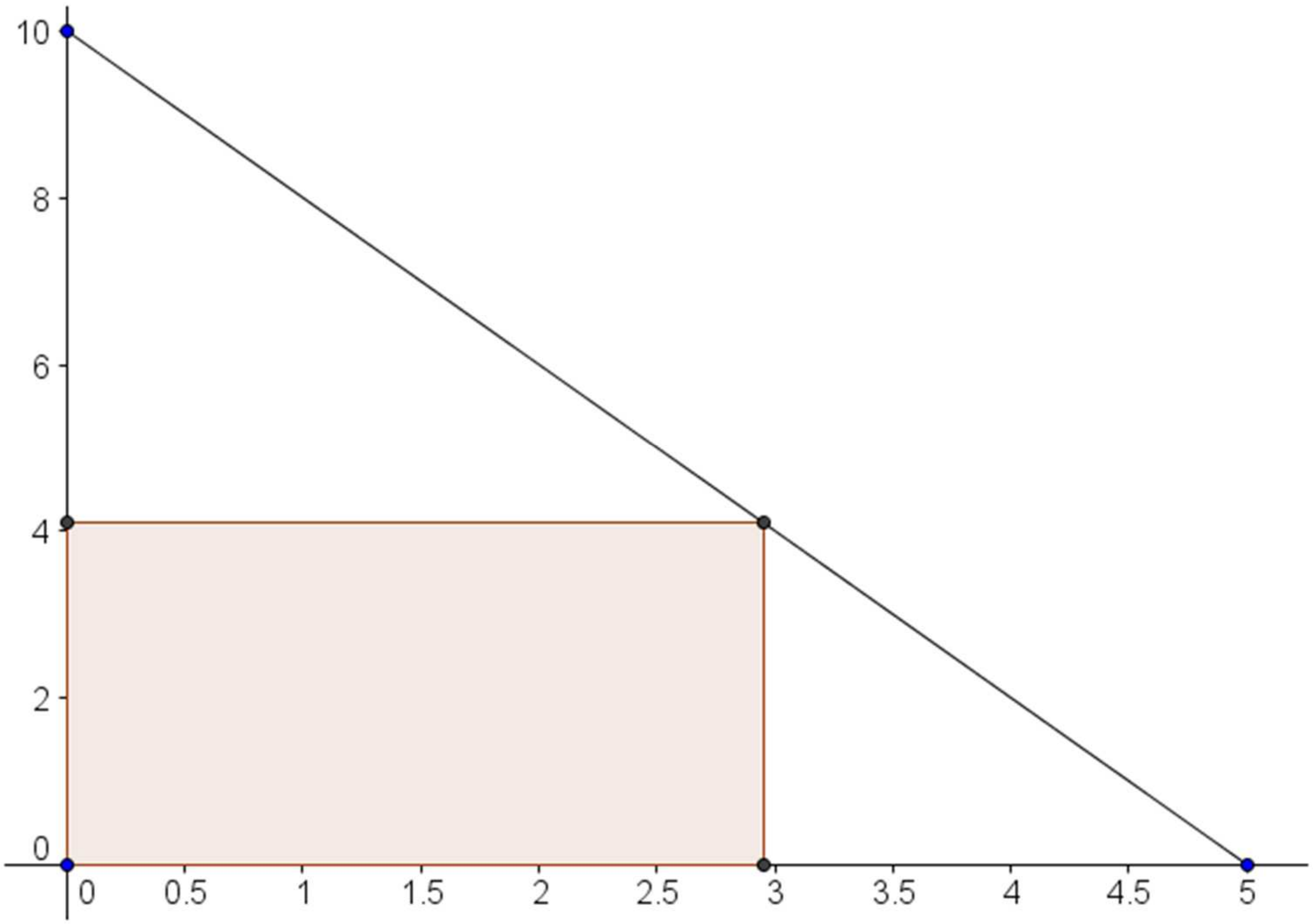
The function $f(x) = 1 - \frac{1}{8}x^2$ is graphed below. Use basic shapes to approximate the shaded area. Your answer should include both a sketch and a numeric value.



A Maximization Problem

A rectangle with sides parallel to the x and y axes has its lower left hand vertex at the origin and its upper right hand vertex in the first quadrant along the line $y = 10 - 2x$. Give the dimensions of the rectangle so that it has the largest possible area.

Maximization Figure



A Related Probability Problem

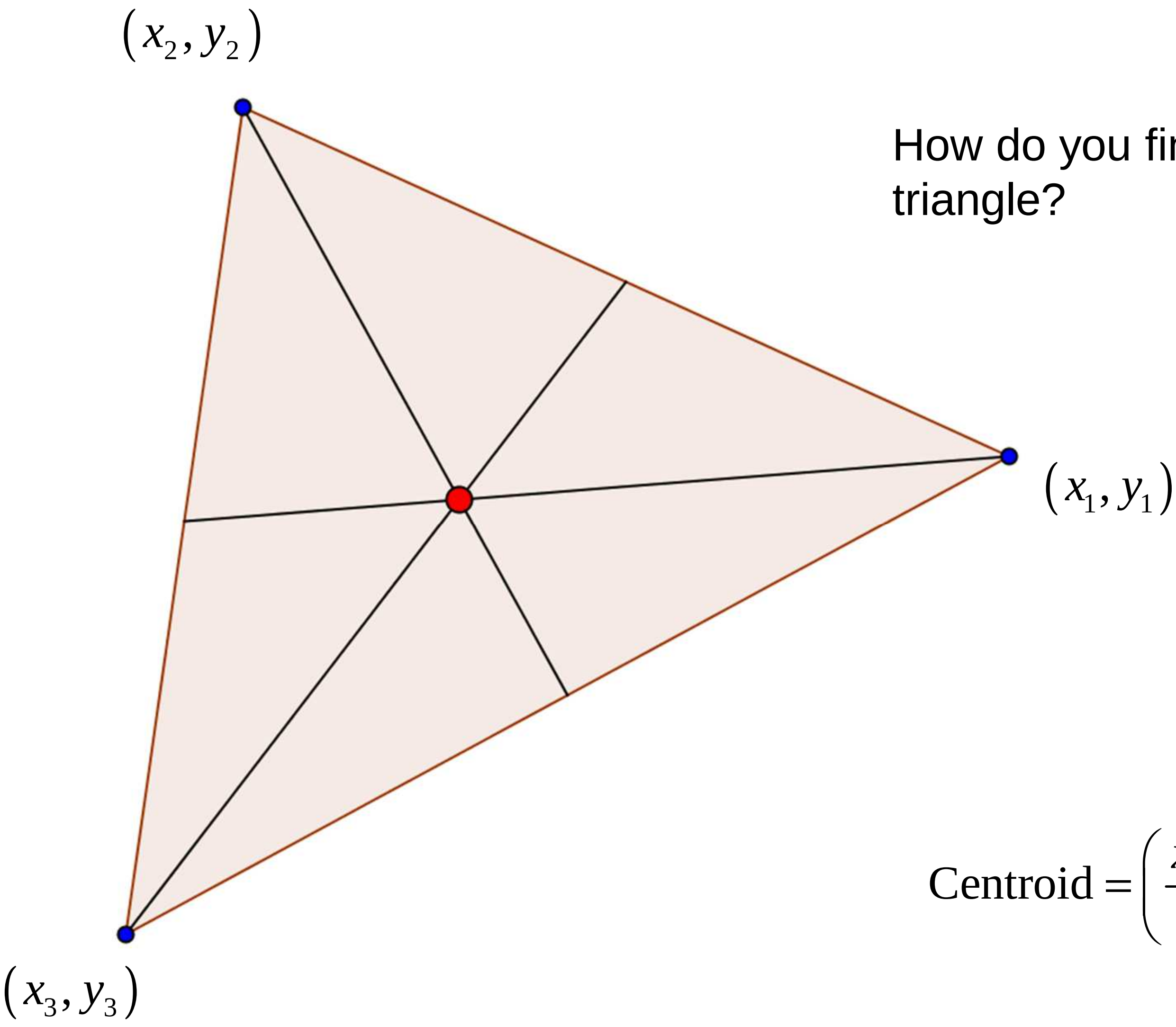
A rectangle with sides parallel to the x and y axes has its lower left hand vertex at the origin and its upper right hand vertex in the first quadrant along the line $y = 10 - 2x$. If the rectangle is drawn by randomly selecting a possible lower right vertex, what is the probability the area is larger than 4?

A Crazy Word Problem

Do your students determine what they are being asked before they start attacking a problem?

A crane has a boom that is 40 feet long and is currently extended so that its bottom end is 8 feet off of the ground and forms a 60^o angle with the horizontal. The base of the crane weighs 250 tons, and the crane is cabable of moving in the forward or reverse direction at speeds of 4 mph. The wind speed is 10 mph and the cable on the the crane is 120 feet long, with a portion of the cable wound on a spindle that is 8 feet in diameter. A 500 horse power motor turns a gear with a ratio of 5:1 to turn the spindle. Suppose a loaded boxcar weighs 80 tons and is sitting in an indentation in the ground 20 feet below the base level of the crane. The cable is attached to the boxcar and the crane operator starts to slowly wind the cable to lift the boxcar. If the tensile strength of the cable is sufficient to lift the boxcar and the crane operator chews 3 packs of gum each day, then how many packs will she chew during a 5 day work week?

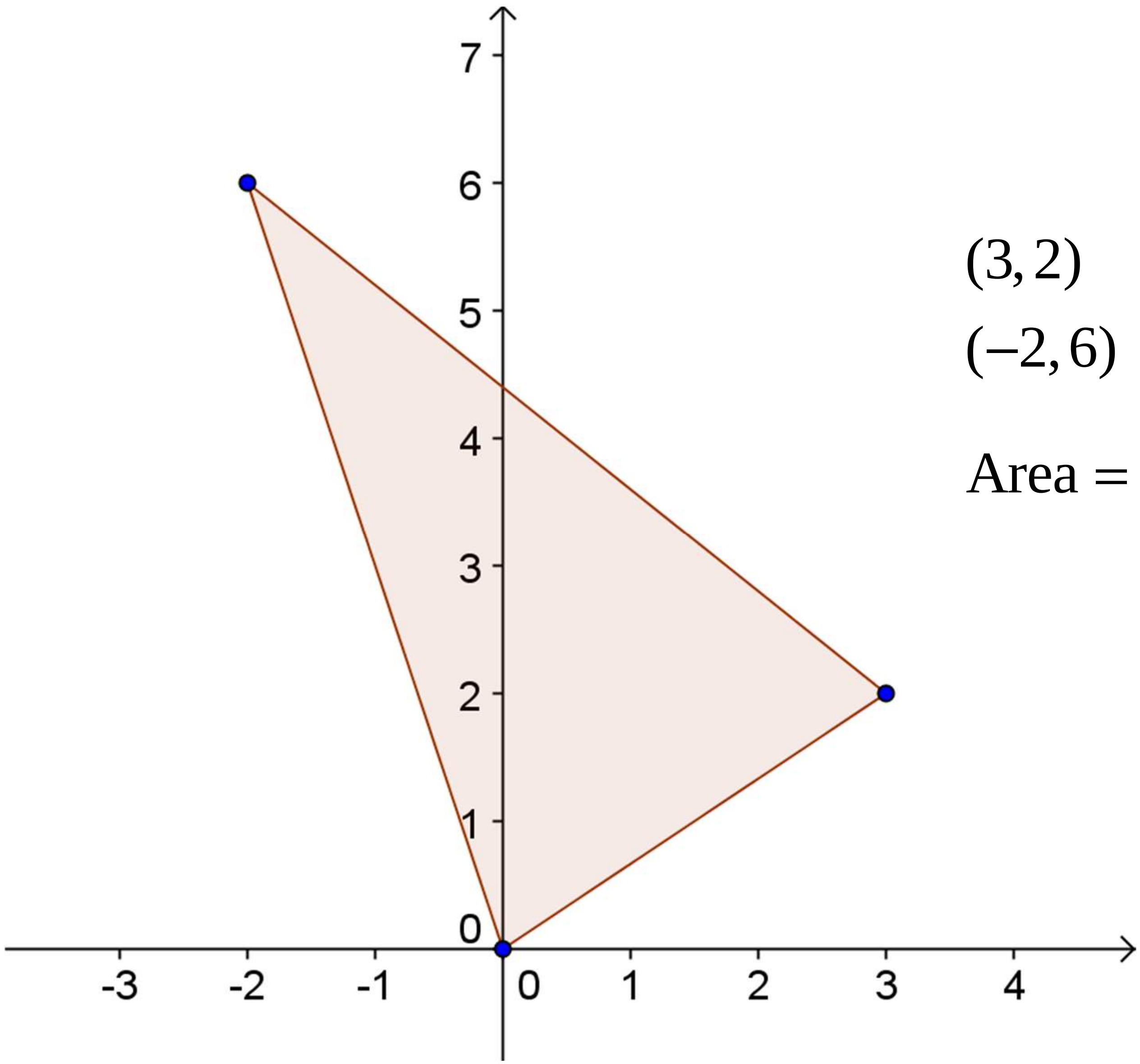
Centroids of Triangles



How do you find the centroid of a triangle?

$$\text{Centroid} = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$$

Areas of Triangles



$$(3, 2)$$

$$(-2, 6)$$

$$\text{Area} = \frac{1}{2}(3 \cdot 6 - 2 \cdot (-2)) = 11$$

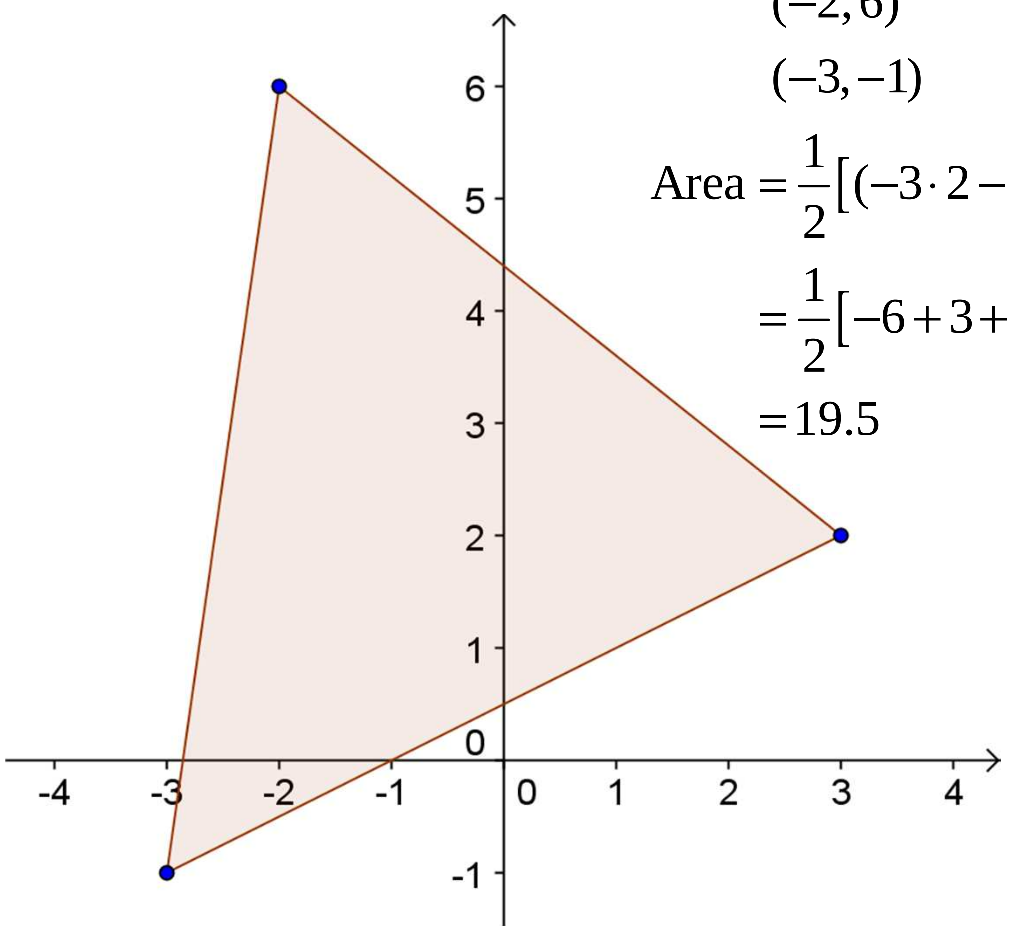
Areas of Triangles

List the vertices ***counter clockwise*** from first to first. It does not matter which vertex you list as "first".

$$(-3,-1)$$
$$(3,2)$$
$$(-2,6)$$
$$(-3,-1)$$

Notice the cross multiplication in the formula below from point to point.

$$\text{Area} = \frac{1}{2}\left[(-3\cdot 2-(-1)\cdot 3)+(3\cdot 6-2\cdot(-2))+(-2\cdot(-1)-6\cdot(-3))\right]$$
$$= \frac{1}{2}\left[-6+3+18+4+2+18\right]$$
$$= 19.5$$



Notice the difference between this triangle and the one in the previous example with a vertex at the origin.

Areas of Triangles

List the vertices ***counter clockwise*** from first to first. It does not matter which vertex you list as "first".

(x_1, y_1)

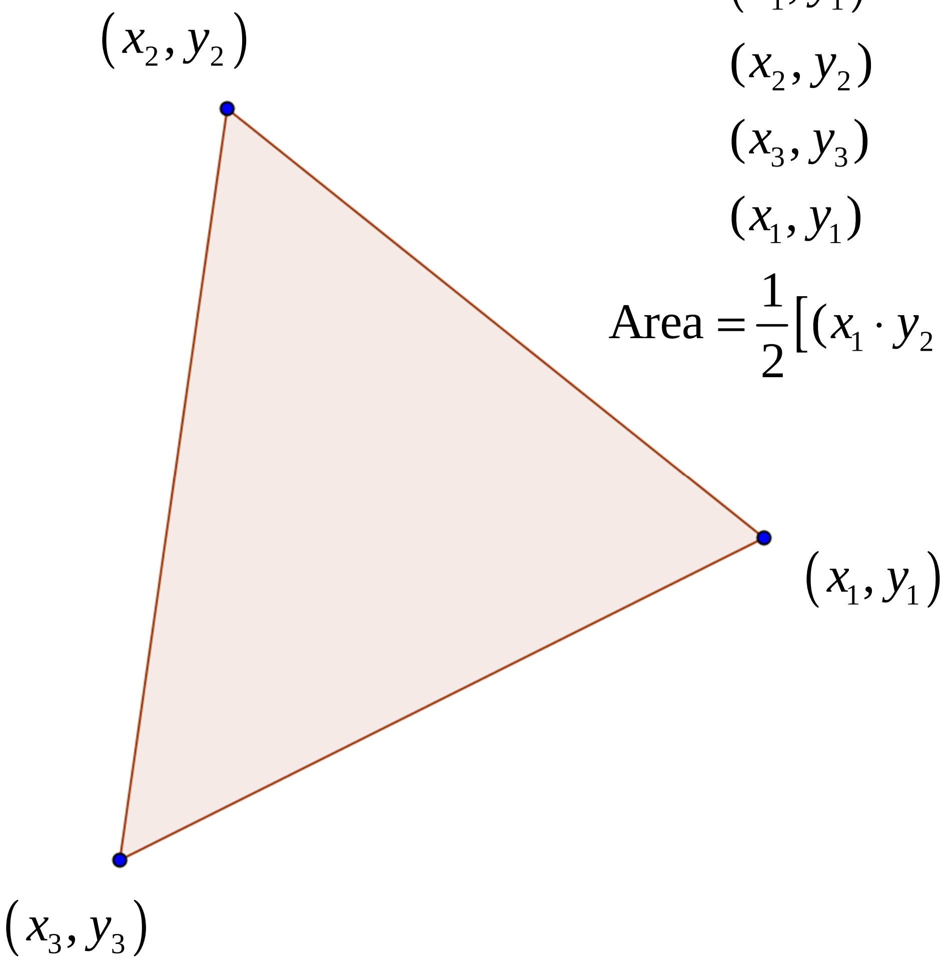
(x_2, y_2)

(x_3, y_3)

(x_1, y_1)

Notice the cross multiplication in the formula below from point to point.

$$\text{Area} = \frac{1}{2}\left[(x_1 \cdot y_2 - y_1 \cdot x_2) + (x_2 \cdot y_3 - y_2 \cdot x_3) + (x_3 \cdot y_1 - y_3 \cdot x_1)\right]$$

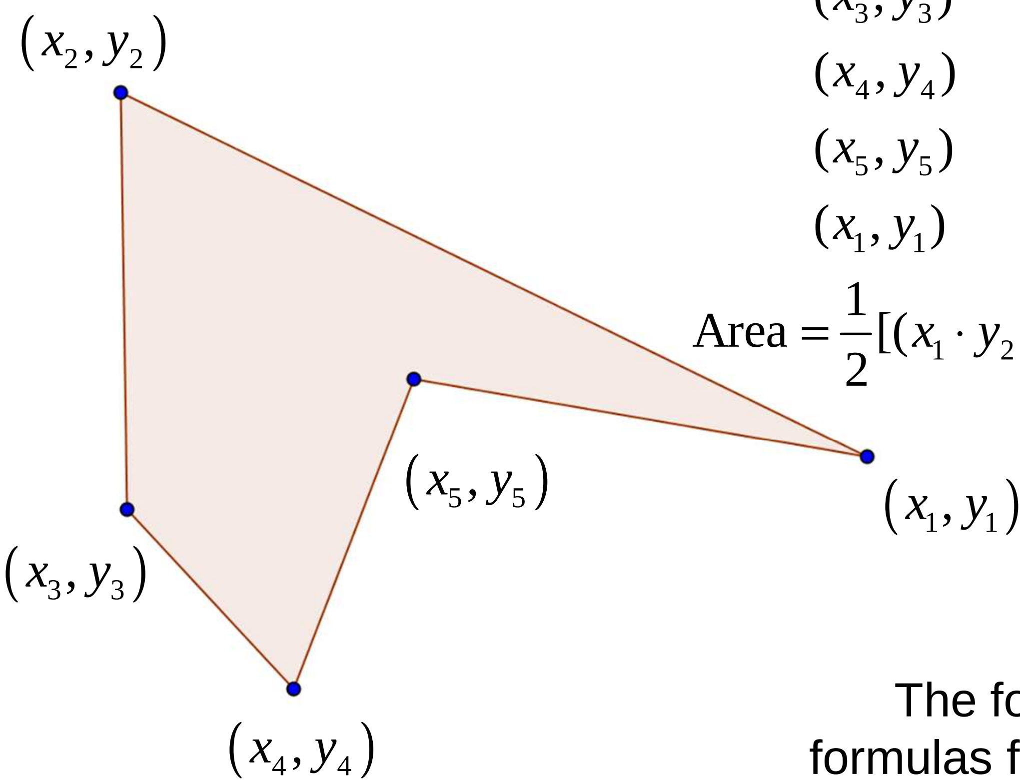


Areas of Polygons

List the vertices ***counter clockwise*** from first to first. It does not matter which vertex you list as “first”.

$$(x_1, y_1)$$
$$(x_2, y_2)$$
$$(x_3, y_3)$$
$$(x_4, y_4)$$
$$(x_5, y_5)$$
$$(x_1, y_1)$$

Notice the cross multiplication in the formula below from point to point.



$$\text{Area} = \frac{1}{2}[(x_1 \cdot y_2 - y_1 \cdot x_2) + (x_2 \cdot y_3 - y_2 \cdot x_3) + (x_3 \cdot y_4 - y_3 \cdot x_4) + (x_4 \cdot y_5 - y_4 \cdot x_5) + (x_5 \cdot y_1 - y_5 \cdot x_1)]$$

The formula can be generalized to give formulas for areas of polygons with an arbitrary number of vertices.