

Integrals Involving Powers of Sine and Cosine

1. If the power of the **sine is odd and positive**, save one sine factor to be the du and convert the remaining factors to cosine. Then, expand and integrate.
2. If the power of the **cosine is odd and positive**, save one cosine factor to be the du and convert the remaining factors to sine. Then, expand and integrate.
3. If the **powers of both the sine and cosine are even and nonnegative**, make repeated use of the half angle identities.

Integrals involving Secants and Tangents

1. If the power of the **secant is even and positive**, save a secant squared factor to be the du and convert the remaining factors to tangents. Then, expand and integrate.
2. If the power of the **tangent is odd and positive**, **and there is a secant factor**, save a secant-tangent factor to be the du and convert the remaining factors to secants. Then, expand and integrate.

$$\int \tan^3 x \sec x \, dx$$

If there is **no secant factor**, convert a tangent squared factor to a secant squared factor; then expand and repeat if necessary.

$$\int \tan^3 x \, dx$$

3. If there are **no secant factors** and the **power of the tangent is even and positive**, convert a tangent squared factor to a secant squared factor; then expand and repeat if necessary.

$$\int \tan^4 x \, dx$$

4. If the integral is of the form $\int \sec^m x dx$ where **m is odd and positive**, use integration by parts.

$$\int \sec^3 x dx$$

5. If **none of the first four cases applies**, try converting to sines and cosines.

$$\int \frac{\sec x}{\tan^2 x} dx$$

Integrals involving Sine-Cosine Products with Different Angles

Use the product-to-sum identities.

$$\sin A \sin B = \frac{1}{2} (\cos(A - B) - \cos(A + B))$$

$$\sin A \cos B = \frac{1}{2} (\sin(A - B) + \sin(A + B))$$

$$\cos A \cos B = \frac{1}{2} (\cos(A - B) + \cos(A + B))$$

Where the product to sum formula comes from and why it may be needed:

Example:

$$\int \sin(Ax) \sin(Bx) dx$$

Recall

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

So,

$$\sin A \sin B = \frac{1}{2} (\cos(A-B) - \cos(A+B))$$

Start of Review for Exam 1 (all mixed up)

$$\arcsin\left(\sin\frac{-\pi}{6}\right)=$$

$$\arctan\left(\tan\frac{-\pi}{4}\right)=$$

$$\cos\left(\arcsin\frac{3}{5}\right)=$$

$$\sec\left(\arctan\frac{4}{3}\right)=$$

$$\tan\left(\operatorname{arcsec}\frac{5}{3}\right) =$$

$$\tan\left(\arcsin\frac{2}{7}\right) =$$

$$y = (\sinh ax)(\cosh ax)$$

$$y' =$$

$$\frac{d}{dx}(\cos x)^{x^2+1} =$$

$$\frac{d}{dx} (3x^2 + 1)^{4x+2}$$

$$\int e^{2x} dx =$$

$$\int 3^{5x} dx =$$

$$\frac{d}{dx} e^{x^2+3x} =$$

$$\frac{d}{dx} 5^{x^2+3x} =$$

$$\frac{d}{dx} 3^{2x} =$$

$$\frac{d}{dx} \ln x =$$

$$\int \frac{\sec^2 x}{\tan x + 1} dx$$

$$\int \frac{\sec^2 3x}{1 + \tan 3x} dx$$

$$\frac{d}{dx} \arctan(5x) =$$

$$\frac{d}{dx} \arcsin(3x^2) =$$

$$\int \tan^4 x \sec^2 x \, dx$$

$$\int \frac{1}{a^2 + u^2} \, du =$$

$$\int \frac{u}{a^2 + u^2} \, du =$$

$$\int \frac{2}{\sqrt{a^2 - u^2}} \, du =$$

$$\int \frac{5u}{\sqrt{a^2 - u^2}} du =$$

$$\int \frac{7}{225 + (3x - 1)^2} dx =$$

$$\int \frac{2e^{\frac{1}{x}}}{x^2} dx$$

$$\int \frac{3}{\sqrt{7 - x^2}} dx =$$

$$\int x^2 \ln x \, dx =$$

$$\int 2x \sqrt{2x-3} \, dx =$$

$$\int x \cos 2x \, dx =$$