December 15, 2009

Name:

Final Math 3336

This final is worth **200** points. Each problem is worth **20** points. You are not allowed to use any books or notes.

1. Solve for the integers modulo 7:

$$\begin{aligned} 2x - 3y &= 2 \\ x + y &= 4 \end{aligned}$$

2. Prove that $2^n > n^2$ if n is an integer greater than 4.

3. Let F be the function such that $F(n)$ is the sum of the first n positive integers. Give a recursive definition of $F(n)$.

4. Define the equivalence relation $\sim_f$ for a function $f : A \to B$. In particular, do this for the function $f : \{1, 2, \cdots, 20\} \to \mathbb{N}$ where $f(k)$ is the number of factors in the unique prime factorization of k. For example, $f(8) = 3$. What are the equivalence classes for the associated equivalence relation?

5. (a) Let R be a relation on the set A. How is $R \circ R$ defined?
   (b) Prove that R is transitive in case that $R \subseteq R \circ R$.

6. Find the smallest equivalence relation on the set $\{a, b, c, d, e\}$ containing the relation $R = \{(a, b), (a, c), (d, e)\}$

7. Define that $\leq$ is a partial order on the set A. Give an example of a partial order which is not total.

8. Does the set $\mathbb{N}$ of non-negative natural numbers with divisibility as partial order have a maximum and/or a minimum? Explain!

9. (a) Define that $\mathbb{L}$ is a lattice.
   (b) Explain why the set $\mathbb{N}$ with divisibility as order is a lattice.

10. Let E and F be equivalence relations on the set $A = \{a, b, c, d, e, f, g, h\}$ where the partition for E is given by

$$\pi_E = \{\{a, g, h\}, \{b, c, d\}, \{e\}, \{f, g\}\}$$

and the partition for F is

$$\pi_F = \{\{a, f, g\}, \{b, d, g, h\}, \{c, e\}\}$$

Find the partition of $E \vee F$ and $E \wedge F$.