September 25, 2010

Test 1 Math 5331

You have **90 minutes** to complete the test. You cannot use any books, notes or calculators.

1. Let a and b be real numbers. Find all vectors y in $\mathbb{R}^2$ which are perpendicular to $x = (a, b)$.

2. Find all solutions of the system of equations:

$$\begin{array}{ccccccc} x & + & y & + & z & = 1 \\ x & - & y & - & z & = 1 \end{array}$$

3. Solve over the complex numbers

$$\begin{array}{ccccc} x & - & iy & = & 0 \\ ix & + & 2y & = & -1 \end{array}$$

4. (a) Let

$$\mathbf{A} = \begin{pmatrix} 2 & 1 & 2 \\ 3 & 1 & 4 \\ 7 & 3 & 8 \end{pmatrix}$$

   Find the row-echelon form $\mathbf{B}$ of $\mathbf{A}$.

   (b) For which choices of A, B and C is the inhomogeneous system consistent?

$$\begin{array}{ccccccc} 2x & + & y & + & 2z & = A \\ 3x & + & y & + & 4z & = B \\ 7x & + & 3y & + & 8z & = C \end{array}$$

5. Find the inverse of the matrix:

$$\begin{pmatrix} 1 & a & b & c \\ 0 & 1 & d & e \\ 0 & 0 & 1 & f \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

6. Prove that for any a_i, b_j, where not all a_i are 0 and not all b_j are 0, the matrix $\mathbf{A}$ has rank 1:

$$\mathbf{A} = \begin{pmatrix} a_1b_1 & a_1b_2 & \cdots & a_1b_n \\ a_2b_1 & a_2b_2 & \cdots & a_2b_n \\ \cdots\cdot & \cdots\cdot & \cdots & \cdots\cdot \\ a_nb_1 & a_nb_2 & \cdots & a_nb_n \end{pmatrix}$$

7. Can a linear inhomogeneous system with real entries have a solution over the field $\mathbb{C}$ of complex numbers but not over the field $\mathbb{R}$ of real numbers? Explain your answer.

8. Let $\mathbf{A}$ and $\mathbf{B}$ be both $n \times n-$matrices. Prove that the product $\mathbf{C} = \mathbf{AB}$ is invertible if and only if $\mathbf{A}$ and $\mathbf{B}$ are invertible.
   Hint: Use that a matrix $\mathbf{C}$ is invertible if and only $\mathbf{C}x = 0$ has only the trivial solution.