November 6, 2010

Test 2 Math 5331

Each problem is worth **20** points. You cannot use any books, notes or calculators.

1. Which of the following subsets of $\mathbb{R}^3$ are subspaces?
   (a) $\{(x,y,z) \mid x+y=0 \text{ and } y+z=0\}$
   (b) $\{(x,y,z) \mid x=0 \text{ or } y=0\}$
   (c) $\{(x,y,z) \mid x=1\}$
   (d) $\{(x,y,z) \mid x=y \text{ and } z=0\}$

2. For the vector $v=(1,-1,1,-1,1) \in \mathbb{R}^5$ find a linear system $AX=0$ whose solution space is the span of v.

3. Find a basis of the solution space of
$$\begin{pmatrix} 1 & -1 & 2 & 0 & 2 & 1 \\ 1 & 1 & 2 & 0 & 2 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{pmatrix} = 0$$

4. For the vectors $\alpha_1 = (1,0,2,2)$, $\alpha_2 = (1,1,0,3)$, $\alpha_3 = (1,4,0,0)$ find a linear homogeneous system which has the span of these vectors as solution space.

5. (a) Decide whether the vector $(4,11,12)$ is in the span of $u=(1,2,3)$ and $v=(2,5,6)$. You have to justify your answer.
   (b) Find an equation for the plane generated by the vectors u and v

6. Prove that $u=(1,0,0)$, $v=(1,1,0)$ and $r=(1,1,1)$ form a basis of $\mathbb{R}^3$. Let T be the linear map where $T(u)=(2,1,3)$, $T(v)=(1,1,2)$, $T(r)=(0,0,1)$. What is $T(6,5,3)$?

7. The following is a bonus problem. What is the dimension of the vector space of all symmetric $n \times n$ matrices?