Name: **December 4, 2010**

FINAL

Math 5331 Linear Algebra Online

This Final is worth 200 points. You are not allowed to use any books or notes. You have three hours to complete the test.

1. Define that $\mathbf{U}$ is a subspace of the vector space $\mathbf{V}$.

2. Proof that the solution space of a linear homogeneous system $\mathbf{A}X = 0$ of m equations in n unknowns with real coefficients is a subspace of $\mathbb{R}^n$

3. (a) Define that the vectors $v_1, v_2, \ldots, v_k$ are linearly independent.
   (b) Define the span of vectors $v_1, v_2, \ldots, v_k$.

4. Define that the vectors $v_1, v_2, \ldots, v_k$ are a basis of the vector space $\mathbf{V}$.

5. Find a basis of all solutions for the linear homogeneous system

$$\begin{pmatrix} 1 & -1 & 0 & 5 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{pmatrix} = 0$$

6. If $\mathbf{A}$ is an $m \times n$ matrix of rank r, what is then the dimension of the solution space of the linear homogeneous system $\mathbf{A}X = 0$?

7. (a) Define that $T : \mathbf{U} \to \mathbf{V}$ is a linear map from the vector space $\mathbf{U}$ to the vector space $\mathbf{V}$.
   (b) How are nullspace and range of a linear map defined?
   (c) Let T be a linear map from $\mathbb{R}^n$ to $\mathbb{R}^m$. How is the matrix $\mathbf{A}$ for T with respect to the unit vectors defined?
   (d) Express null space and range of T in terms of the matrix $\mathbf{A}$ for T. In particular relate column and row rank of $\mathbf{A}$ to the dimensions of null space and range of T.

8. Let $T : \mathbf{U} \to \mathbf{V}$ be a linear map from the vector space $\mathbf{U}$ to the vector space $\mathbf{V}$. Prove that if the vectors $T(v_1), T(v_2), \ldots, T(v_k)$ are linearly independent, then $v_1, v_2, \ldots, v_k$ are linearly independent.

9. Assume that the linear map T on $\mathbb{R}^3$ has matrix

$$\mathbf{A} = \begin{pmatrix} 2 & 1 & 5 \\ 3 & 1 & 7 \\ 1 & 3 & 5 \end{pmatrix}$$

   (a) Find a basis for the null space of T.
   (b) Find a basis for the range of T.

10. Let T be a linear map from $\mathbb{R}^n \to \mathbb{R}^n$ and let $\lambda \in \mathbb{R}$.
   (a) Define the characteristic polynomial $c_T(x)$ of T. What is the characteristic polynomial of the identity map on $\mathbb{R}^n$ and what is the characteristic polynomial of the zero map on $\mathbb{R}^n$?
   (b) Define that λ is an eigenvalue of T.
   (c) Define that $\mathbf{E}_\lambda$ is the eigenspace for λ.