April 2, 2011

Name:

Test 2 Abstract Algebra Math 5330

You have **90** minutes to complete the test. You cannot use any books or notes.

1. Which of the direct products are cyclic? Explain your answers.
   (a) $\mathbb{Z}_2 \times \mathbb{Z}_3 \times \mathbb{Z}_5$.
   (b) $\mathbb{Z}_2 \times \mathbb{Z}_2$.
   (c) $\mathbb{Z} \times \mathbb{Z}$.
2. Calculate the order of (8,6,4) in $\mathbb{Z}_{18} \times \mathbb{Z}_9 \times \mathbb{Z}_8$.
3. Let $f : A \to B$ and $g : B \to C$ be maps such that $g \circ f : A \to C$ is injective (i.e., one-to-one). Prove that f must be injective.
4. Find for the function $f : \mathbb{N} \to \mathbb{N}$, where $f(n) = 2n, n = 1, 2, \ldots$, some function $g : \mathbb{N} \to \mathbb{N}$ such that $g \circ f$ is the identity on $\mathbb{N}$. Can you find some h such that $f \circ h$ is the identity on $\mathbb{N}$?
5. Find the right cosets of the subgroup $< (1, 1) >$ in $\mathbb{Z}_2 \times Z_4$
6. Let $\mathbf{G}$ be any group and $x \in \mathbf{G}$. Let σ be the map $\sigma : y \mapsto xyx^{-1}$. Prove that this map is bijective.
7. (a) Let R be an equivalence relation on the set S, and let $s \in S$. How is the equivalence class of s under R defined?
   (b) Let R be the equivalence relation on the set $\mathbb{R}$ of real numbers where $r \sim s$ iff $|r| = |s|$. What is the equivalence class of r?
8. (a) Find the order of the following permutation in S_{10}:
   $$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 3 & 4 & 5 & 2 & 1 & 7 & 8 & 6 & 10 & 9 \end{pmatrix}$$
   (b) Is this permutation even or odd?
9. Let p be a prime and G a group whose order is p. Prove that G is cyclic.
10. Let G be a group and let H and K be subgroups of G where $|H|$ and $|K|$ are relatively prime. Prove that $H \cap K = \{e\}$.