

Name:

Test 1 Math 3336

You have the full class period to complete the test. You cannot use any books or notes.

This test is worth 250 points.

1. 40 pts.

Prove or disprove whether the formula is a tautology or not:

- (a) $(p \rightarrow q) \vee (q \rightarrow p)$
- (b) $(p \rightarrow q) \vee (p \rightarrow \neg q)$
- (c) $((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$
- (d) $(\neg p \wedge (p \rightarrow q)) \rightarrow \neg q$

2. 30 pts.

Express $((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$ in Polish notation and draw the formation tree.

3. 30 pts. But each wrong answer carries a penalty of -5 pts.

Mark as true or false. The implication *If Q, then P* is equivalent to:

- a) P is sufficient for Q.
- b) Q is sufficient for P.
- c) P is necessary for Q.
- d) Q is necessary for P.
- e) P if Q.
- f) Q only if P.

4. 30 pts. But each wrong answer carries a penalty of -5 pts.

Determine whether the following arguments are valid or invalid.

- (a) If dogs purr, then cats bark. Dogs don't purr. Thus cats don't bark.
- (b) If dogs purr, then cats bark. Cats don't bark. Thus dogs don't purr.
- (c) Dogs purr if cats bark. Cats don't bark. Thus dogs don't purr.

5. 50 pts.

Find the conjunctive and disjunctive normal form for the proposition $P(p_1, p_2, p_3)$ which has truth table:

p_1	p_2	p_3	$P(p_1, p_2, p_3)$
T	T	T	T
T	T	F	T
T	F	T	F
F	T	T	F
T	F	F	T
F	T	F	T
F	F	T	F
F	F	F	T

6. 30 pts.

Decide whether the following formulas are equivalent. In case where your answer is "not equivalent" you must give an explanation.

- (a) $\exists x(Q(x) \wedge P(x))$ and $\exists xQ(x) \wedge \exists xP(x)$
- (b) $\exists x(Q(x) \vee P(x))$ and $\exists xQ(x) \vee \exists xP(x)$
- (c) $\exists x(Q(x) \rightarrow P(x))$ and $\exists xQ(x) \rightarrow \exists xP(x)$

7. **40 pts.**

Formalize the following statement: Every student in this class has a favorite teacher whom he likes to take any class from, unless it has the word "abstract" in the title. (Hint: Use "x" for students in this class, "y" for UH teachers, "z" for courses at UH, $L(x,y,z)$ "x likes to take course z from (his favorite teacher) y, $A(z)$ course z has the word abstract in the title.)