## Practice Sheet for Test 3 Math 3336

- 1. Prove by induction that 6 divides  $n^3 n$  whenever n is a positive integer. '
- 2. Prove that a set with  $n \ge 2$  elements has n(n-1)/2 subsets containing exactly 2 elements.
- 3. Give a recursive definition of  $x^n$  for a real number x.
- 4. Give a recursive definition of the sequence  $a_n = n(n+1)$ .
- 5. (a) Define that R is an equivalence relation on the set A.
  - (b) Define that  $\pi$  is a partition of the set A.
  - (c) Describe the partition  $\pi_R$  for an equivalence R.
  - (d) Define the equivalence  $R_{\pi}$  for a partition  $\pi$ .
- 6. Let  $f: A \to B$  be any function from the set A to the set B. How is the equivalence relation  $\sim_f$  on A defined?
- 7. Let  $f : \mathbb{R} \to \mathbb{R}, x \mapsto x^2$ , be the parabola function. What does the partition for the equivalence relation of this function look like?
- 8. Determine which of the following relations on the set  $\mathbb{Z}$  of integers are equivalence relations.
  - (a) aRb iff  $a b \ge 0$ .
  - (b) aRb iff  $ab \ge 0$ .
  - (c) aRb iff |a| = |b|.
  - (d) aRb iff  $|a-b| \le 1$ .
- 9. Prove that the intersection of two equivalence relations E and F is an equivalence relation. Explain why the union of two equivalence relations is in general not an equivalence relation. Explain the meaning  $E \vee F$ .
- 10. Let E and F be equivalence relations on the set  $A = \{a, b, c, d, e, f, g\}$  where the partition for E is given by

$$\pi_E = \{\{a\}, \{b, c, d\}, \{e\}, \{f, g\}\}\$$

and the partition for F is

$$\pi_F = \{\{a, f\}, \{b, d, g\}, \{c, e\}\}$$

Find the partition of  $E \vee F$  and  $E \wedge F$ .