Test 2 Math 3336

You have the full class period to complete the test. You cannot use any books or notes. Problems 1-11 are each worth **20** points. A false answer in problem 1 gives you **-5** points.

- 1. Mark as true or false.
 - (a) A function is injective if f(a) = f(b) in case that a = b.
 - (b) A function is injective if $a \neq b$ yields $f(a) \neq f(b)$.
 - (c) A function is injective if f(a) = f(b) only if a = b.
 - (d) A function is injective only when $f(a) \neq f(b)$ yields $a \neq b$.
- 2. Show that the map $f: A \to B$ is injective in case that there is some map $g: B \to A$ such that $g \circ f = id_A$.
- 3. Find an injective map f on the set \mathbb{N} of natural numbers which is injective but not surjective. Can you find a map on a finite set F which is surjective but not injective.
- 4. Define the relation a|b (a divides b) between integers a and b and then define the greatest common divisor, gcd(a, b), and the lowest common multiple, lcm(a, b). Is there a number m for which you have n|m for every n.
- 5. State the Division algorithm for natural numbers.
- 6. Prove that $ab = \gcd(a, b) \cdot \operatorname{lcm}(a, b)$.
- 7. Let k > 0. Define that $n \equiv m \mod (k)$.
- 8. Convert 200 from decimal notation to base 3 notation.
- 9. (a) State the Chinese Remainder Theorem.
 (b) Find some x such that x ≡ 8 mod (18), x ≡ 9 mod (19)
- 10. (a) Find the inverse of 12 modulo 41.
 - (b) Solve $2x = 7 \mod (13)$
- 11. (a) Define that an integer is prime.
 - (b) Define that an integer is irreducible.