February 14, 2015

Name:

Test 1 Math 5330

You have **90** minutes to complete the test. You cannot use any books or notes.

1. State the Well-Ordering Principle for $\mathbb{Z}^+$.
2. Prove by induction that $1 + 3 + 5 + \cdots + (2n + 1) = (n + 1)^2$.
3. Prove by Mathematical Induction that if a set S has n elements then S has 2^n−many subsets.
4. Is division a commutative operation on the set $\mathbb{R}^+$ of positive numbers? Is it associative? Explain your answers.
5. Define that $(G, *)$ is a group. You can also state that $(G, *, ^{-1}, e)$ is a group.
6. Define the additive group $(\mathbb{Z}_n, \oplus)$ of integers modulo n. You have to state exactly what its elements are, how $\oplus$ is defined, what the identity is, and how the inverse of an element is defined.
7. (a) Find the additive inverse of 40 modulo 48.
   (b) Solve $15 + x = 7$ modulo 48.
8. Let $(G, *)$ be a group such that $x^2 = e$ for all $x \in G$. Show that G is abelian.
9. Let G be a finite group, and consider the multiplication table for multiplication of G. Prove that every element of G occurs precisely once in each row and once in each column.
10. Find a multiplication table on the set $A = \{a, b, c, d\}$ where every element of A occurs precisely once in each row and once in each column but where A is **not** a group. You have to explain why your algebra $A = (\{a, b, c, d\}, *)$ is not a group.