Practice sheet for Test 2.

1. Mark as true or false.

a. A function is injective if $f(a) = f(b)$ only if $a = b$.

b. A function is injective if $a = b$ yields $f(a) = f(b)$.

c. A function is injective if $f(a) \neq f(b)$ only if $a \neq b$.

d. A function is injective if $f(a) \neq f(b)$ in case that $a \neq b$.

2. Let A be a set and $\mathsf{P}(A)$ be the power set of A. Mark as true or false.

a. There is an injection from A to $\mathsf{P}(A)$.

b. There is a surjection from A to $\mathsf{P}(A)$.

3. Find a bijection from the set N of natural numbers to the set E of even natural numbers.

4. Use the Cantor-Bernstein Theorem in order to prove that there is a bijection from the open interval $(0, 1)$ to the closed interval $[0, 1]$.

5. Determine whether each of these statements are true or false.

a) $\emptyset \in \emptyset$ b) $\emptyset \in \{\{\emptyset\}\}$ c) $\emptyset \subseteq \{\emptyset\}$

d) $\{\emptyset\} = \{\emptyset, \emptyset\}$ e) $\{\emptyset\} \subseteq \{\emptyset, \emptyset\}$ f) $\{\emptyset\} \subseteq \{\{\emptyset\}\}$

6. What is the successor of the set $\{adam, eve\}$?

7. Determine whether the function $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ is onto if

a) $f(m, n) = m + n$, b) $f(m, n) = m - n$ c) $f(m, n) = m^2 + n^2$

8. Let f be a function from A to B. Let S and T be subsets of A and U and V be subsets of B. True or false:

a) $f(S \cup T) = f(S) \cup f(T)$, b) $f(S \cap T) = f(S) \cap f(T)$……

c) $f^{-1}(U \cup V) = f^{-1}(U) \cup f^{-1}(V)$ d) $f^{-1}(U \cap V) = f^{-1}(U) \cap f^{-1}(V)$

9 Assume for sets A and B that the power sets are equal, that is $P(A) = P(B)$. Can you conclude that $A = B$?

10. a)Is the empty set $\emptyset$ the power set of a set? b)Is $\{\emptyset, \{a\}, \{b\}, \{a, b\}\}$ the power set of a set?