Practice sheet for Test 2.

1. Mark as true or false.

a. A function is injective if $f(a) = f(b)$ only if $a = b$. **T**

b. A function is injective if $a = b$ yields $f(a) = f(b)$. **F**

c. A function is injective if $f(a) \neq f(b)$ only if $a \neq b$. **T**

d. A function is injective if $f(a) \neq f(b)$ in case that $a \neq b$. **T**

2. Let A be a set and $\mathsf{P}(A)$ be the power set of A. Mark as true or false.

a. There is an injection from A to $\mathsf{P}(A)$. T

b. There is a surjection from A to $\mathsf{P}(A)$. **F**

3. Find a bijection from the set N of natural numbers to the set E of even natural numbers. **Answer**: $n \mapsto 2n$

4. Use the Cantor-Bernstein Theorem in order to prove that there is a bijection from the open interval $(0,1)$ to the closed interval $[0,1]$. **Answer**: **We first need an injection from** $(0,1)$ to $[0,1]$. T**he inclusion will do it**. **And we need an injection from** $[0,1]$ **to** $(0,1)$. **For doing this we may map bijectively** $[0,1]$ **to a closed subinterval of** $[0,1]$, **say** $[\frac{1}{4}, \frac{1}{2}]$. **The linear map** $y = \frac{1}{4}x + \frac{1}{4}$ **will do this**. **Then followed by the inclusion of** $[\frac{1}{4}, \frac{1}{2}]$ **to** $(0,1)$ **yields an injection from** $[0,1]$ **to** $(0,1)$.

5. Determine whether each of these statements are true or false. Answers:

a) $\emptyset \in \emptyset$ **F** b) $\emptyset \in \{\{\emptyset\}\}$ **F** c) $\emptyset \subseteq \{\emptyset\}$ **T**

d) $\{\emptyset\} = \{\emptyset, \emptyset\}$ **T** e) $\{\emptyset\} \subseteq \{\emptyset, \emptyset\}$ **T** f) $\{\emptyset\} \subseteq \{\{\emptyset\}\}$ **F**

6. What is the successor of the set $\{adam, eve\}$? **Answer**: $\{adam, eve, \{adam, eve\}\}$

7. Determine whether the function $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ is onto if,

a) $f(m,n) = m + n$, **T** b) $f(m,n) = m - n$ **T** c) $f(m,n) = m^2 + n^2$ **F**

8. Let f be a function from A to B. Let S and T be subsets of A and U and V be subsets of B. True or false:

a) $f(S \cup T) = f(S) \cup f(T)$, **T** b) $f(S \cap T) = f(S) \cap f(T)$ **F**……

c) $f^{-1}(U \cup V) = f^{-1}(U) \cup f^{-1}(V)$ **T** d) $f^{-1}(U \cap V) = f^{-1}(U) \cap f^{-1}(V)$ **T**

9 Assume for sets A and B that the power sets are equal, that is $P(A) = P(B)$. Can you conclude that $A = B$? **Answer**: **Yes**, $A \in P(A) = P(B)$, **thus** $A \subseteq B$. **Similarly**, $B \in P(B) = P(A), B \subseteq A$. **Thus** $A = B$.

10. a) Is the empty set $\emptyset$ the power set of a set? **NO**, **any powerset contains** $\emptyset$ b) Is $\{\emptyset, \{a\}, \{b\}, \{a,b\}\}$ the power set of a set?

Yes, $\{\emptyset, \{a\}, \{b\}, \{a,b\}\} = P\{a,b\}$