Practice sheet for Math 3336 Fall 2016

October 24, 2016

- 1. Define for integers *a* and *b* the relation a|b. Prove that 1|a and a|0.
- **2**. Let *a* and *b* be integers and let *m* be a positive integer. Define that *a* is congruent to *b* modulo *m*. What are the elements congruent to 0? Prove that every integer *a* is congruent mod *m* to a unique $0 \le r < m$.
- 3. Evaluate these quantities.
 - **a**. 13 mod 3
 - **b**. -97 mod 11
 - **c**. 155 mod 19
 - **d**. -221 mod 23
- 4. Convert the decimal expansion of each of these integers to a binary expansion.
 - **a**. 22
 - **b**. 100
 - **c**. 60
 - **d**. 9
- 5. Express the greatest common divisor of each of these pairs of integers as a linear combination of these integers.
 - **a**. 10,11
 - **b**. 9,16
 - **c**. 0,20
 - **d**. 99,101
- 6. Find all invertible elements and their inverses in
 - **a**. (ℤ₁₀, •)
 - **b**. (\mathbb{Z}_{11}, \cdot)
- 7. Solve mod 5 the linear equation 2x + 3 = 1
- 8. Which integers leave a remainder 1 when divided by 2 and also leave a remainder 1 when divided by 3.