

December 1, 2016

### Practice Test 5 and Final

For your final, the first ten problems on equivalence relations and partial orders count as Test 5. The second part is comprehensive and more theoretical. It will have five questions.

#### Practice problems

- Which of these relations on the set  $A$  of all people are equivalence relations.
  - $\{(a,b)|a \text{ and } b \text{ have met}\}$
  - $\{(a,b)|a \text{ and } b \text{ are the same age}\}$
  - $\{(a,b)|a \text{ and } b \text{ speak Spanish}\}$
  - $\{(a,b)|a \text{ and } b \text{ are not the same nationality}\}$
- Let  $f: A = \{1,2,3,4\} \rightarrow B = \{a,b,c\}$  be the map  $1 \mapsto c, 2 \mapsto b, 3 \mapsto c, 4 \mapsto a$ . Find the partition  $\pi_E$  for the equivalence relation  $E = \ker(f)$ .
- Find  $R \circ R$  for the relation  $R = \{(a,c), (a,d), (b,a), (b,b), (d,a), (d,b)\}$ .
- Find the smallest equivalence relation on the set  $\{a,b,c,d,e\}$  containing the relation  $\{(a,b), (a,c), (d,e)\}$ .
- List the ordered pairs in the equivalence relation  $E_\pi$  for the partition  $\{\{a,b\}, \{c,d\}, \{e,f,g\}\}$  of the set  $A = \{a,b,c,d,e,f,g\}$
- Which of these are posets?
  - $(\mathbb{Z}, =)$
  - $(\mathbb{Z}, \neq)$
  - $(\mathbb{Z}, >)$
  - $(\mathbb{Z}, |)$
- Which of these relations on the set  $A$  of all people are posets?
  - $a$  is taller than  $b$ .
  - $a$  is not taller than  $b$
- Find maximal and minimal elements of the poset  $A = (\{1, 3, 5, 9, 15, 24, 45\}, |)$ .
- Find a compatible order for the poset  $A$  of the preceding problem.
- Find all incompatible pairs of elements in the poset  $A$  of the preceding problem 8.

#### Practice Final Questions

- Let  $f: A \rightarrow B$  be a function. How is the equivalence kernel  $E_f$  for  $f$  defined?
- Define that  $u$  is an upper bound for the subset  $S$  of the poset  $(A, \leq)$ .
- Give the definition that the poset  $(L, \leq)$  is a lattice.
- How are the integers mod  $n$  defined? How many elements has  $\mathbb{Z}_n$  and how

are addition and multiplication defined?

5. Define the divisibility relation  $|$  on the set  $\mathcal{N}$  of natural numbers .