December 1, 2016

Practice Test 5 and Final

For your final, the first ten problems on equivqalence relations and partial orders count as Test 5. The second part is comprehensive and more theoretical. It will have five questions.

Practice problems

1. Which of these relations on the set A of all people are equivalence relations.

(**a**) $\{(a,b)|a$ and b have met$\}$ **Answer**: Not an equivalence. Not transitive.

(**b**) $\{(a,b)|a$ and b are the same age$\}$ **Answer**: Equivalence.

(**c**) $\{(a,b)|a$ and b speak Spanish$\}$ **Answer**: Equivalence

(**d**) $\{(a,b)|a$ and b are not the same nationality$\}$**Answer**: Not an equivalence. Not reflexive and not transitive.

2. Let $f: A = \{1,2,3,4\} \to B = \{a,b,c\}$ be the map $1 \mapsto c, 2 \mapsto b, 3 \mapsto c, 4 \mapsto a$. Find the partition π_E for the equivalence relation $E = \ker(f)$. Answer: $\pi = \{\{1,3\},\{2\},\{4\}\}$

3. Find $R \circ R$ for the relation $R = \{(a,c),(a,d),(b,a),(b,b),(d,a),(d,b)\}$. **Answer**: $R \circ R = \{(a,a),(a,b),(b,c),(b,d),(b,a),(b,b),(d,c),(d,d),(d,a),(d,b)\}$

4. Find the smallest equivalence relation on the set $\{a,b,c,d,e\}$ containing the relation $\{(a,b),(a,c),(d,e)\}$. **Answer**: The partition π for this equivalence is $\pi = \{\{a,b,c\},\{d,e\}\}$

5. List the ordered pairs in the equivalence relation E_π for the partition $\{\{a,b\},\{c,d\},\{e,f,g\}\}$ of the set $A = \{a,b,c,d,e,f,g\}$
Answer: $E = (a,a),(b,b),(a,b),(b,a),(c,c),(d,d),(c,d),(d,c),(e,e),(f,f),(g,g),(e,f),(f,e$

6. Which of these are posets?

(**a**) $(\mathbb{Z},=)$ **Answer**: yes

(**b**) $(\mathbb{Z},\neq)$**Answer**:No

(**c**) $(\mathbb{Z},>)$**Answer**:No,not reflexive

(**d**) $(\mathbb{Z},|)$ **Answer**: yes

7. Which of these relations on the set A of all people are posets?

(**a**) a is taller than b. **Answer**: No, not reflexive.

(**b**) a is not taller than b **Answer**: yes. It is the relation $a \leq b$ where $>$ indicates taller, thus $\neg(a > b)$ is the same as $a \leq b$.

8. Find maximal and minimal elements of the poset $A = (\{1,3,5,9,15,24,45\},|)$. **Answer**: 1 is the only minimal element, actually it is the minimum. Maximal elements are $45, 24$

9. Find a compatible order for the poset A of the preceding problem. **Answer**: $1 < 5 < 3 < 9 < 15 < 24 < 45$

10. Find all incompatible pairs of elements in the poset A of the preceding problem 8. **Answer**: $(3,5),(5,24).(5,9),(5,24),(9,15),(9,24),(15,24),(24,45)$

Practice Final Questions

1. Let $f : A \to B$ be a function. How is the equivalence kernel E_f for f defined? **Answer**: aEb iff $f(a) = f(b)$

2. Define that u is an upper bound for the subset S of the poset $(A, \leq)$. **Answer**: $u \geq s$ for all $s \in S$.

3. Give the definition that the poset $(L, \leq)$ is a lattice. **Answer**: Any two elements a, b have a least upper bound, called $a \vee b$ and a largest lower bound, called $a \wedge b$.

4. How are the integers $\bmod n$ defined? How many elements has $\mathbb{Z}_n$ and how are addition and multiplication defined? **Answer**: $a \equiv_n b$ iff $n|b - a$ is an equivalence relation on the set of $\mathbb{Z}$ of integers. $\mathbb{Z}_n$ is the set of equivalence classes. There are n –many equivalence classes corresponding to n –many remainders if a number is divided by $n : \mathbb{Z}_n = \{[0]_n, [1]_n, \ldots, [n-1]_n\}$. We can add classes: $[a]_n + [b]_n = [a+b]_n, [a]_n \cdot [b]_n = [a \cdot b]_n$

5. Define the divisibility relation | on the set N of natural numbers . **Answer**: $n|m$ iff $\exists_k \; k \cdot n = m$