You have the full class period to complete the test. You cannot use any books or notes.

This test is worth 250 points.

1. 40 pts.
Prove or disprove whether the formula is a tautology or not:
   a. \((p \rightarrow q) \lor (q \rightarrow p)\)
   b. \((p \rightarrow q) \lor (p \rightarrow \neg q)\)
   c. \(((p \lor q) \land (\neg p \lor r)) \rightarrow (q \lor r)\)
   d. \((\neg p \land (p \rightarrow q)) \rightarrow \neg q\)
   e. \((p \rightarrow (q \rightarrow r)) \leftrightarrow (p \land q) \rightarrow r\).

2. 30 pts.
Express \((p \rightarrow (q \rightarrow r))\) in Polish notation and draw the formation tree.

30 pts. But each wrong answer carries a penalty of -5 pts.
Mark as true or false. The implication \(If Q, then P\) is equivalent to:
   a) P is sufficient for Q.
   b) Q is sufficient for P.
   c) P is necessary for Q.
   d) Q is necessary for P.
   e) P if Q.
   f) Q only if P.

3. 30 pts. But each wrong answer carries a penalty of -5 pts.
Determine whether the following arguments are valid or invalid.
   a. Only hard working people make good money. Paul is not hard working. Thus Paul does not make good money.
   b. Only hard working people make good money. Paul makes good money. Thus Paul is hard working.
   c. Only hard working people make good money. Paul does not make good money. Thus Paul is not hard working.

4. 40 pts.
Find the conjunctive and disjunctive normal form for \(p \leftrightarrow q\).

5. 30 pts.
Decide whether the following formulas are equivalent. In case where your answer is "not equivalent" you must give an explanation.
   a. \(\exists x(Q(x) \land P(x))\) and \(\exists xQ(x) \land \exists xP(x)\)
   b. \(\exists x(Q(x) \lor P(x))\) and \(\exists xQ(x) \lor \exists xP(x)\)
   c. \(\exists x(Q(x) \rightarrow P(x))\) and \(\exists xQ(x) \rightarrow \exists xP(x)\)
6. **50 pts.**

Let $L(x, y)$ be the predicate “the student $x$ likes the course $y$”, $H(x, y)$ the predicate “$x$ works hard for course $y$”, and $G(x, y)$ the predicate “$x$ makes a good grade for course $y$”. Then formalize:

a. Some students like course $y = c$.

b. Every student likes some course $y$.

c. There is a course every student likes.

d. Unless a student works hard for a certain course $y = c$, he won’t make a good grade in that course.

e. Only students who like course $y = c$ work hard for $y = c$. 