September 19, 2013

Name:

You have the full class period to complete the test. You cannot use any books or notes.

This test is worth 250 points.

1. **40 pts**.
   Prove or disprove whether the formula is a tautology or not:
   - **a**. $(p \to q) \vee (q \to p)$
   - **b**. $(p \to q) \vee (p \to \neg q)$
   - **c**. $((p \vee q) \wedge (\neg p \vee r)) \to (q \vee r)$
   - **d**. $(\neg p \wedge (p \to q)) \to \neg q$
   - **e**. $(p \to (q \to r)) \leftrightarrow (p \wedge q) \to r$.
2. **30 pts**.
   Express $(p \to (q \to r))$ in Polish notation and draw the formation tree.

   30 pts. **But each wrong answer carries a penalty of -5 pts**.
   Mark as true or false. The implication *If Q, then P* is equivalent to:

a) P is sufficient for Q.	b) Q is sufficient for P.
c) P is necessary for Q.	d) Q is necessary for P.
e) P if Q.	f) Q only if P.

3. **30 pts**. **But each wrong answer carries a penalty of -5 pts**.
   Determine whether the following arguments are valid or invalid.
   - **a**. Only hard working people make good money. Paul is not hard working. Thus Paul does not make good money.
   - **b**. Only hard working people make good money. Paul makes good money. Thus Paul is hard working
   - **c**. Only hard working people make good money. Paul does not make good money. Thus Paul is not hard working.
4. **40 pts**.
   Find the conjunctive and disjunctive normal form for $p \leftrightarrow q$.
5. **30 pts**.
   Decide whether the following formulas are equivalent. In case where your answer is "not equivalent" you must give an explanation.
   - **a**. $\exists x(Q(x) \wedge P(x))$ and $\exists xQ(x) \wedge \exists xP(x)$
   - **b**. $\exists x(Q(x) \vee P(x))$ and $\exists xQ(x) \vee \exists xP(x)$
   - **c**. $\exists x(Q(x) \to P(x))$ and $\exists xQ(x) \to \exists xP(x)$

6. **50 pts**.

Let $L(x,y)$ be the predicate "the student x likes the course y", $H(x,y)$ the predicate "x works hard for course y", and $G(x,y)$ the predicate "x makes a good grade for course y". Then formalize:

a. Some students like course $y = c$.

b. Every student likes some course y.

c. There is a course every student likes.

d. Unless a student works hard for a certain course $y = c$, he won't make a good grade in that course.

e. Only students who like course $y = c$ work hard for $y = c$.