This test is worth 250 points.

1. **40 pts.**
   Prove or disprove whether the formula is a tautology or not:
   a. \((p \rightarrow q) \lor (q \rightarrow p)\) **Answer:** Tautology. Done in class
   b. \((p \rightarrow q) \lor (p \rightarrow \neg q)\) **Answer:** Tautology. Done in class
   c. \((p \lor q) \lor (\neg p \lor r)\) **Answer:** Tautology. Done in class
   d. \((p \lor (p \rightarrow q))\) **Answer:** Not a tautology: \(p = F, q = T\) makes \((p \lor (p \rightarrow q)) = T\) but \((p \lor (p \rightarrow q)) \rightarrow \neg q = F\)
   e. \((p \rightarrow (q \rightarrow r)) \leftrightarrow (p \lor q) \rightarrow r. **Answer:** Tautology.

2. **30 pts.**
   Express \((p \rightarrow (q \rightarrow r))\) in Polish notation and draw the formation tree. The polish notation is \(\rightarrow p \rightarrow qr\)
   **30 pts.**
   Mark as true or false. The implication *If \(Q\), then \(P\)* is equivalent to:
   *If \(Q\), then \(P\)* is the same as \(Q \rightarrow P\)
   a) \(P\) is sufficient for \(Q\) \(F\) b) \(Q\) is sufficient for \(P\) \(T\)
   c) \(P\) is necessary for \(Q\) \(T\) d) \(Q\) is necessary for \(P\) \(F\)
   e) \(P\) if \(Q\) \(T\) f) \(Q\) only if \(P\) \(T\)

3. **30 pts.**
   Determine whether the following arguments are valid or invalid. Let \(H(p)\) stand for \(p\) is hard working, \(M(p)\) stand for \(p\) is making good money.
   a. Only hard working people make good money. Paul is not hard working. Thus Paul does not make good money. Answer: Only hard working people make good money is \(M(p) \rightarrow H(p)\), Paul is not hard working is \(\neg H(p)\). So \(\neg M(p)\). Correct argument.
   b. Only hard working people make good money. Paul makes good money. Thus Paul is hard working. **Answer:** \(M(p) \rightarrow H(p), M(p)\). So \(H(P)\). Correct.
   c. Only hard working people make good money. Paul does not make good money. Thus Paul is not hard working. **Answer:** \(M(p) \rightarrow H(p), \neg M(p)\) does not yield \(\neg H(p)\) Paul
might be on bad terms with his supervisor. \textbf{Invalid}.

4. 40 pts.
Find the conjunctive and disjunctive normal form for $p \leftrightarrow q$. DNF: $(p \land q) \lor (\neg p \land \neg q)$
CNF: $(\neg p \lor q) \land (p \lor \neg q)$

5. 30 pts.
Decide whether the following formulas are equivalent. In case where your answer is "not equivalent" you must give an explanation.
\begin{itemize}
\item[a.] $\exists x (Q(x) \land P(x))$ and $\exists x Q(x) \land \exists x P(x)$ Not equivalent.
$Q(x) \equiv x > 0, P(x) \equiv x < 0$. Right hand side is valid for integers, left hand side is not.
\item[b.] $\exists x (Q(x) \lor P(x))$ and $\exists x Q(x) \lor \exists x P(x)$ Equivalent.
\item[c.] $\exists x (Q(x) \land P(x))$ and $\exists x Q(x) \land \exists x P(x)$ Not equivalent. Let
$U = \{a, b\}, Q(a), \neg Q(b), \neg P(a), P(b)$. Then $\exists x Q(x) = T$ namely for $x = a$ and
$\exists x P(x) = T$ namely for $x = b$. But $\exists x (Q(x) \land P(x)) = F$. there is no common $x$ that satisfies $Q(x)$ and $P(x)$.
\end{itemize}

6. 50 pts.
Let $L(x, y)$ be the predicate “the student $x$ likes the course $y$”, $H(x, y)$ the predicate “$x$ works hard for course $y$”, and $G(x, y)$ the predicate “$x$ makes a good grade for course $y$”. Then formalize:
\begin{itemize}
\item[a.] Some students like course $y = c$. \textbf{Answer:} $\exists x L(x, c)$
\item[b.] Every student likes some course $y$. \textbf{Answer:} $\forall x \exists y L(x, y)$
\item[c.] There is a course every student likes. \textbf{Answer:} $\exists y \forall x L(x, y)$
\item[d.] Unless a student works hard for a certain course $y = c$, he won’t make a good grade in that course. \textbf{Answer:} $\forall x (G(x, c) \rightarrow H(x, c))$
\item[e.] Only students who like course $y = c$ work hard for $y = c$. \textbf{Answer:} $\forall x (H(x, c) \rightarrow L(x, c))$
\end{itemize}