September 19, 2013

Name:

You have the full class period to complete the test. You cannot use any books or notes.

This test is worth 250 points.

1. **40 pts**.
Prove or disprove whether the formula is a tautology or not:
   - **a**. $(p \to q) \vee (q \to p)$ **Answer**:
   $(\neg p \vee q) \vee (\neg q \vee p) \equiv (q \vee \neg q) \vee (\neg p \vee p) \equiv T \vee T \equiv T$ One could also argue In order to make $(p \to q) \vee (q \to p)$ false one needs to make $(p \to q)$ false and $(q \to p)$ false that is, $p = T, q = F$ and $q = T, p = F$ which is impossible. **It is a tautology**
   - **b**. $(p \to q) \vee (p \to \neg q)$ **Answer**: **Tautology**. Done in class
   - **c**. $((p \vee q) \wedge (\neg p \vee r)) \to (q \vee r)$ **Answer**: **Tautology**. In order to make the formula false, we need $(p \vee q) = T, (\neg p \vee r) = T, (q \vee r) = F$. But then $q = F, r = F, p = T$ and $(\neg p \vee r)$ cannot be made T
   - **d**. $(\neg p \wedge (p \to q)) \to \neg q$ **Answer**: **Not a tautology**: $p = F, q = T$ makes $(\neg p \wedge (p \to q)) = T$ but $(\neg p \wedge (p \to q)) \to \neg q = F$
   - **e**. $(p \to (q \to r)) \leftrightarrow (p \wedge q) \to r$. **Answer**: **Tautology**.
   $(p \to (q \to r)) \equiv \neg p \vee (q \to r) \equiv \neg p \vee \neg q \vee r \equiv \neg(p \wedge q) \vee r \equiv (p \wedge q) \to r$
2. **30 pts**.
Express $(p \to (q \to r))$ in Polish notation and draw the formation tree. The polish notation is $\to p \to qr$

   30 pts.
   Mark as true or false. The implication *If Q, then P* is equivalent to:
   If Q, then P is the same as $Q \to P$

   a) P is sufficient for Q.**F** b) Q is sufficient for P.**T**

   c) P is necessary for Q.**T** d) Q is necessary for P.**F**

   e) P if Q.**T** f) Q only if P **T**
3. **30 pts**.
Determine whether the following arguments are valid or invalid. Let *H(p) stand for p is hard working, M(p) stand for p is making good money.*
   - **a**. Only hard working people make good money. Paul is not hard working. Thus Paul does not make good money. Anwer: Only hard working people make good money is $M(p) \to H(p)$, Paul is not hard working is $\neg H(p)$. So $\neg M(p)$. Correct argument.
   - **b**. Only hard working people make good money. Paul makes good money. Thus Paul is hard working. **Anwer**: $M(p) \to H(p), M(p)$. So $H(P)$. Correct.
   - **c**. Only hard working people make good money. Paul does not make good money. Thus Paul is not hard working. **Answer**: $M(p) \to H(p), \neg M(p)$ does not yield $\neg H(p)$ Paul

might be on bad terms with his supervisor. **Invald**.

4. **40 pts**.
Find the conjunctive and disjunctive normal form for $p \leftrightarrow q$. DNF: $(p \wedge q) \vee (\neg p \wedge \neg q)$ CNF; $(\neg p \vee q) \wedge (p \vee \neg q)$

5. **30 pts**.
Decide whether the following formulas are equivalent. In case where your answer is "not equivalent" you must give an explanation.
   - **a**. $\exists x(Q(x) \wedge P(x))$ and $\exists x Q(x) \wedge \exists x P(x)$ **Not equivalent**. $Q(x) \equiv x > 0, P(x) \equiv x < 0.$ Right hand side is valid for integers, left hand side is not.
   - **b**. $\exists x(Q(x) \vee P(x))$ and $\exists x Q(x) \vee \exists x P(x)$ **Equivalent**.
   - **c**. $\exists x(Q(x) \wedge P(x))$ and $\exists x Q(x) \wedge \exists x P(x)$ **Not equivalent**. Let $U = \{a,b\}, Q(a), \neg Q(b), \neg P(a), P(b).$ Then $\exists x Q(x) = T$ namely for $x = a$ and $\exists x P(x) = T$ namely for $x = b.$ But $\exists x(Q(x) \wedge P(x)) = F.$ there is no common x that satisfies $Q(x)$ and $P(x)$.

6. **50 pts**.
Let $L(x,y)$ be the predicate “the student x likes the course y”, $H(x,y)$ the predicate “x works hard for course y”, and $G(x,y)$ the predicate “x makes a good grade for course y”. Then formalize:
   - **a**. Some students like course $y = c$. **Answer**: $\exists x L(x,c)$
   - **b**. Every student likes some course y. **Answer**: $\forall x \exists y L(x,y)$
   - **c**. There is a course every student likes. **Answer**: $\exists y \forall x L(x,y)$
   - **d**. Unless a student works hard for a certain course $y = c$, he won’t make a good grade in that course. **Answer**: $\forall x(G(x,c) \rightarrow H(x,c))$
   - **e**. Only students who like course $y = c$ work hard for $y = c$. **Answer**: $\forall x(H(x,c) \rightarrow L(x,c))$