**September 13, 2016**

**Name**:

You have the full class period to complete the test. You cannot use any books or notes.

**This test is worth 300 points**.

1. **60 pts**.
   Prove or disprove whether the formula is a tautology or not:
   a. $(\neg p \wedge (p \longrightarrow q)) \rightarrow \neg q$ **Answer**: **Not a tautology**. $p = F, q = T$ makes $(\neg p \wedge (p \longrightarrow q)) \rightarrow \neg q = \mathbf{F}$
   b. $(\neg q \wedge (p \longrightarrow q)) \rightarrow \neg p$ **Answer**: **Tautology**. **To make** $(\neg q \wedge (p \longrightarrow q)) \rightarrow \neg p$ **false you need to make** $(\neg q \wedge (p \longrightarrow q)) = T$ **and** $\neg p = F$, **that is** $p = T$. **Now** $(\neg q \wedge (p \longrightarrow q)) = T$ **needs** $\neg q = T$, **that is** $q = F$ **but then** $(p \longrightarrow q)$ **needs** $p = F$. **Thus** $(\neg q \wedge (p \longrightarrow q)) \rightarrow \neg p$ **cannot be made false**.
   c. $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$ **Answer**: **Tautology**. **In order to make** $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$ **false we need to make** $[(p \rightarrow q) \wedge (q \rightarrow r)] = T$ **and** $(p \rightarrow r) = F$, **that is** $p = T$ **and** $r = F$. **Now** $[(p \rightarrow q) \wedge (q \rightarrow r)] = T$ **gives us** $(p \rightarrow q) = T$ **and** $(q \rightarrow r) = T$. **But we already have** $p = T$, **thus we get** $q = T$ **and then** $r = T$. **This contradicts** $r = F$.
   d. $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$ **Answer**:**Tautology**. **The formula is** $F$ **if** $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] = T$ **and** $r = F$. **But then** $(p \rightarrow r) \wedge (q \rightarrow r) = T$ **requires** $p = F$ **and** $q = F$ **and we get** $(p \vee q) = F$ **and** $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] = T$ **is impossible**.
2. **60 pts**.
   Mark as true or false. The implication *P only if Q* is equivalent to: **Answers** *P only if Q* $\equiv P \rightarrow Q$

   a) P is sufficient for Q. **T** b) Q is sufficient for P.**F**

   c) P is necessary for Q. **F** d) Q is necessary for P. **T**

   e) P if Q. **F** f) Q if P. **T**
3. **30 pts**.
   Determine whether the following arguments are valid or invalid. **Answers**: *I am going to the movies* $= M$, *It is raining* $= R$;
   a. Only if it is raining I am going to the movies. It is not raining. Thus I am not going to the movies. **Answer**: **Valid**. **We have** $M \rightarrow R$ **which is the same as** $\neg R \rightarrow \neg M$. **Thus** $\neg R$ **yields** $\neg M$.
   b. If it is raining I am going to the movies. It is not raining. Therefore I am not going to the movies. **Answer**: **Invalid**. **We have** $R \rightarrow M$ **and** $\neg R$. **We cannot conclude** $\neg M$. **I might go to the movies for other reasons**.
   c. If it is raining I am going to the movies. I am going to the movies. Therefore it is raining. **Answer**: **Invalid**. **Given** $R \rightarrow M$ **and** $M$ **does not lead to** $R$.
4. **40 pts**.
   Find the conjunctive normal form for $P = [(p \leftrightarrow q) \rightarrow r]$. (Hint: Find out when $P = F$)

**Answer**: $(\neg p \vee \neg q \vee r) \wedge (p \vee q \vee r)$. **To get to this, we need to find the disjunctive normal form for** $\neg P$, **that is where** $P$ **is false. But this is exactly the case when** $(p \leftrightarrow q) = T$ **and** $r = F$. **Thus** $\neg P = (p \wedge q \wedge \neg r) \vee (\neg p \wedge \neg q \wedge \neg r)$. **By DeMorgan we now get** $P = (\neg p \vee \neg q \vee r) \wedge (p \vee q \vee r)$

5. **60 pts**.
   Decide whether the following formulas are equivalent. You don't have to give an explanation.
   a. $\forall x(P(x) \rightarrow Q(x))$ and $\forall xP(x) \rightarrow \forall xQ(x)$ **Answer**: **Not equivalent. If we choose predicates** $P$ **and** $Q$ **for which we have** $\neg P(a)$ **then** $\forall xP(x)$ **is false and regardless of** $Q$, **one has that the RHS** $\forall xP(x) \rightarrow \forall xQ(x)$ **is true. Now, if we choose** $P$ **that** $P(b)$ **holds and for** $Q$ **that** $\neg Q(b)$ **is true then** $P(b) \rightarrow Q(b)$ **is false. Thus** $\forall x(P(x) \rightarrow Q(x))$ **is false.**
   b. $\forall x(P(x) \wedge Q(x))$ and $\forall xP(x) \wedge \forall xQ(x)$. **Answer**: **Equivalent. Both sides say** $Q(a)$ **for every** $a$.
   c. $\forall x(P(x) \vee Q(x))$ and $\forall xP(x) \vee \forall xQ(x)$. **Answer**: **Not equivalent. Let** $P(x)$ **say that** $x$ **is a boy and** $Q(x)$ **say that** $x$ **is a girl . Then** $\forall x(P(x) \vee Q(x))$ **is true but** $\forall xP(x) \vee \forall xQ(x)$ **is not.**
6. **50 pts**.
   Let $L(x,y)$ be the predicate "*x likes to buy the car y*", $K(x,y)$ the predicate "$x$ knows everything about the car $y$", and $M(y)$ the predicate "car $y$ has good milage". Then formalize:
   a. Everybody likes to buy some car. **Answer**: $\forall x \exists y L(x,y)$.
   b. There is some car everybody likes to buy. **Answer**: $\exists y \forall x L(x,y)$
   c. $x = paul$ likes to buy a car $y$ only if it has good milage. **Answer**: $\forall y L(paul,y) \rightarrow M(y)$
   d. $x = paul$ knows everything about car $y = honda$. **Answer**: $K(paul, honda)$
   e. There is a car nobody likes to buy **Answer**: $\exists y \forall x \neg L(x,y)$