Equivalence using quantifiers.
1.4 #43 : Are

$$\forall x(P(x) \to Q(x)) \text{ and } \forall xP(x) \to \forall xQ(x)$$

logically equivalent? Obviously not. Let $P(x)$ stand for "x is German" and $Q(x)$ stand for "x likes Sauerkraut" then

$$\forall x(P(x) \to Q(x)) \text{ stands for "all Germans like Sauerkraut"}$$

while

$$\forall xP(x) \to \forall xQ(x) \text{ stands for "if everybody is German then everybody likes Sauerkraut"}$$

The book gives as answer for this problem: Let $P(x)$ be any propositional function that is sometimes true and sometimes false and let $Q(x)$ be any propositional function that is always false. Then $\forall x(P(x) \to Q(x))$ is false, namely where $P(x)$ is true, but $\forall xP(x) \to \forall xQ(x)$ is true, because $\forall xP(x)$ is false.

To be concrete, let $D = \{a, b\}$ and let a be German and b not German. Both don't like Sauerkraut. Then $\forall x(P(x) \to Q(x))$ is false, at a, but $\forall xP(x) \to \forall xQ(x)$ is true. In order to make $\forall x(P(x) \to Q(x))$ false, we only need that $Q(a)$ is false.

For a more mathematical example, we could chose $D = \mathbb{N} =$ natural numbers, $P(x) = x$ is prime, $Q(x) = x < 0$. Then $\forall xP(x) \to \forall xQ(x)$ is true, because not every number x is prime, and $\forall x(P(x) \to Q(x))$ is false, where x is prime.

If we have that $P(x)$ is always true then $\forall x(P(x) \to Q(x))$ is true in case that $Q(x)$ is true for all x. Then $\forall x(P(x) \to Q(x))$ and $\forall xP(x) \to \forall xQ(x)$ are equivalent, namely to $\forall xQ(x)$. In our German example, being German would be equivalent to liking sauerkraut.

1.4 #45 states that $\exists x(P(x) \vee Q(x))$ and $\exists xP(x) \vee \exists xQ(x)$ are equivalent: $\exists x(P(x) \vee Q(x))$ is true iff for some $x = c$ we have that $P(c)$ or $Q(c)$ is true iff $\exists xP(x)$ or $\exists xQ(x)$ hold.

Are $\exists x(P(x) \wedge Q(x))$ and $\exists xP(x) \wedge \exists xQ(x)$ equivalent. Obviously not. Let $D = \mathbb{N} =$ natural numbers and $P(x) =$ "x is even", $Q(x) =$ "x is odd" . Then $\exists xP(x) \wedge \exists xQ(x)$ is true but $\exists x(P(x) \wedge Q(x))$ is never true.

Propositional equivalences can always be determined by setting up truth tables. A propositional function $f(p_1, \ldots, p_n)$ which is always true, like $f(p) = p \vee \neg p$, is called a tautology. Setting up a truth table for $f(p_1, \ldots, p_n)$ decides whether $f(p_1, \ldots, p_n)$ is a tautology or not. This is the only general method to decide whether a given propositional function is a tautology. If you already know that $f(p_1, \ldots, p_n)$ is a tautology then making $f(p_1, \ldots, p_n) = F$ may quickly yield a contradiction.