Test 2, Math3336 **October 11, 2018**

You have the full class period to complete the test. Every problem is worth 20 points.

1. Mark as true or false.

a. A function f is injective if $f(a) = f(b)$ in case that $a = b$. F

b. A function f is injective if $a = b$ then $f(a) = f(b)$. F

c. A function f is injective if $f(a) \neq f(b)$ then $a \neq b$. F

d. A function f is injective if $a \neq b$ then $f(a) \neq f(b)$. T

2. Let A be a set and $\mathsf{P}(A)$ be the power set of A. Let $f : A \to P(A)$. Mark as true or false.

a. The function f cannot be surjective. T

b. The subset $R = \{a | a \notin f(a)\} \quad \notin im(f)$ T

3. Find a bijection $f : \mathbb{N} \twoheadrightarrow \mathbb{N}$ from the set $\mathbb{N}$ of natural numbers to itself which is not the identity. **Answer**: $f(1) = 2, f(2) = 1, f(n) = n$ for $n > 2$

4. Sketch a function that establishes a bijection between $\mathbb{R}$ and the open interval $(-1, 1)$.
Answer: Any artangent type function will do F

5. Determine whether each of these statements are true or false.
a) $\emptyset \in \{\emptyset\}$ T b) $\emptyset \in \{\{\emptyset\}\}$ F
c) $\{\emptyset\} = \{\emptyset, \{\emptyset\}\}$F d) $\{\emptyset\} \subseteq \{\{\emptyset, \emptyset\}\}$F

6. Define the successor A^+ of the set A. Find the successor of
a)$A = \{\emptyset, \{\emptyset\}\}$ $A^+ = A \cup \{A\} = \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$, b)
$A = \{a\}$ $A^+ = A \cup \{A\} = \{a, \{a\}\}$

7. State the Schröder-Bernstein Theorem. **Answer**:
Theorem Let A and B. be sets. If there is an injection from A to B and an injection from B to A then there is a bijection from A to B.

8. Assume that $f : A \twoheadrightarrow B$ and $g : B \neg A$ are functions such that the composition $g \circ f$ is the identity function on A, that is $g(f(a)) = a$ for every $a \in A$. Prove that a) g is surjective and b) f is injective. **Answer**: ad a) $g(f(a) = a$ Thus $g(b) = a$ for $b = f(a)$ ad b) Assume $f(a_1) = f(a_2)$. Then $g(f(a_1) = g(f(a_2)$, thus $a_1 = a_2$.

9 Let $f(x) = e^x$ and $g(x) = x + 1$. Find $f \circ g$ and $g \circ f$.. **Answer**: Both compositions are functions from $\mathbb{R}$ to $\mathbb{R}$. $f(g(x)) = e^{x+1}$ and $g(f(x) = e^x + 1$

10. a)Is the emty set $\emptyset$ the power set of a set? b)Is $\{\{\emptyset\}, \{\{\emptyset\}\}\}$, the power set of a set? You must prove your answers. **Answer**: a) The emty set cannot be the powerset of any set. Any powerset contains the empty set. b) again, the mpty set is not a member of $\{\{\emptyset\}, \{\{\emptyset\}\}\}$.