February 8, 2018

Test 1, Math 3336

Each of the **10** problems is worth **20** points. You cannot use any books or notes.

1. Translate the given statement into propositional logic using the propositions provided:
You cannot edit a protected Wikipedia entry unless you are an administrator.
Express your answer in terms of e and a where

e: *You can edit a protected Wikipedia entry* **a**: *You are an administrator*

Answer: $\neg a \to \neg e$ which is the same as $e \to a$

*2. (no partial credit)According to US law, you cannot vote unless you are a citizen. This law says:

a. If you are a citizen then you can vote T or F b. You cannot vote if you are not a citizen T or F

Answer: v=you can vote, c=you are a citizen. Law: $v \to c$ or $\neg c \to \neg v$ a is false, b. is true

3. Express these system specifications using the propositions

p: *The message is scanned for viruses* **q**: *The message was sent from an unknown system.*

a. The message is scanned for viruses whenever it was sent from an unknown system Answer: $q \to p$

b. The message was sent from an unknown system but it was not scanned for viruses. Answer: $q \wedge \neg p$

c. It is necessary to scan the message for viruses whenever it was sent from an unknown system Answer: $q \to p$

d. When a message is not sent from an unknown system it is not scanned for viruses. Answer: $\neg q \to \neg p$

4. Using any method, show that the following formula is a tautology: $((p \vee q) \wedge (\neg p \vee r)) \to (q \vee r)$ Answer: In order to make the formula F, $(q \vee r)$ has to be F and $(p \vee q)$ and $(\neg p \vee r)$ have to be T. That is $q = F, r = F,$ but then $p = T$ and $\neg p = T$ which is impossible.

5. Find the disjunctive and conjunctive normal form of $f(p, q, r)$ which is given by the following truth table:

p	q	r	f(p,q,r)
T	T	T	F
T	T	F	F
T	F	T	T
F	T	T	T
T	F	F	T
F	T	F	F
F	F	T	F
F	F	F	T

Answer: DJN:
$(p \wedge \neg q \wedge r) \vee (\neg p \wedge q \wedge r) \vee (p \wedge \neg q \wedge r) \vee (p \wedge \neg q \wedge \neg r) \vee (\neg p \wedge \neg q \wedge \neg r)$ DNF:
$\neg f(p,q,r) = (p \wedge q \wedge r) \vee (p \wedge q \wedge \neg r) \vee (\neg p \wedge q \wedge \neg r) \vee (\neg p \wedge \neg q \wedge r)$ CNF:
$f(p,q,r) = (\neg p \vee \neg q \vee \neg r) \wedge (\neg p \vee \neg q \vee r) \wedge (p \vee \neg q \vee r) \wedge (p \vee q \vee \neg r)$

6. Mark as true or false. The implication *If Q, then P, that is* $Q \rightarrow P$ is equivalent to:

a) P is sufficient for Q. Answer: **F**

b) Q is sufficient for P. Answer: **T**

c) P is necessary for Q. Answer: **T**

d) Q is necessary for P. Answer: **F**

e) P if Q. Answer: **T**

f) Q only if P. Answer: **T**

7. Determine whether the following arguments are valid or invalid. Let *C(p) stand for p is citizen, V(p) stand for p can vote.*

a. Paul is not a citizen. Only citizens can vote. Thus Paul cannot vote. Answer: Correct. $\neg C(paul), V(p) \rightarrow C(p); \neg C(paul) \rightarrow \neg V(p)$

b. Paul is citizen. Only citizens can vote. Thus Paul can vote. Answer: Not correct. $C(paul), V(p) \rightarrow C(p)$ does not yield $C(paul) \rightarrow V(paul)$. You need to be a citizen to vote, but not all citizens can vote.

8. Decide whether the following formulas are equivalent. In case where they are not equivalent you must give an explanation.

a. $\forall x(Q(x) \wedge P(x))$ and $\forall x Q(x) \wedge \forall x P(x)$ Equivalent.

b. $\forall x(Q(x) \vee P(x))$ and $\forall x Q(x) \vee \forall x P(x)$ Not equivalent. $Q(x) \equiv x$ is even, $P(x) \equiv x$ is odd. The left-hand side is true, the right hand side not

c. $\forall x(Q(x) \rightarrow P(x))$ and $\forall x Q(x) \rightarrow \forall x P(x)$ Not equivqlent. Was a homework problem. $Q(x)$ sometimes but not always true, $P(x)$ always false. Then $\forall x Q(x) \rightarrow \forall x P(x)$ is true because $\forall x Q(x)$ is false. However, $\forall x(Q(x) \rightarrow P(x))$ is false for those x where $Q(x)$ is true.

9. Formalize the following statements.

a. For every x there is some y such that x is smaller than y. Answer: $\forall x \exists y(x < y)$

b.For every x and every y there is some z such that z is the sum of x and y. Answer: $\forall x \forall y \exists z(x + y = z)$

c.For every non-negative number x there is some. y such that y^2 is equal to x. Answer: $\forall_{x \geq 0} \exists y \ (y^2 = x)$

d. The absolute value of the product of two integers is the product of their absolute values. Answer: $\forall x \forall y(|xy| = |x||y|)$

10. Let $L(x,y)$ be the statement *x loves y*. Use quantifiers to express each of the statements.

a. Everybody loves somebody. Answer: $\forall x \exists y L(x,y)$

b. There is somebody whom everybody loves.Answer: $\exists y \forall x L(x,y)$

c. There is somebody whom nobody loves. Answer. $\exists y \forall x \neg L(x,y)$

d. Nobody loves everybody. Answer: $\neg(\exists x \forall y(L(x,y)) \ \equiv \forall x \exists y \neg L(x,y)$