Test 2, Math3336 **March 1, 2018**

You have the full class period to complete the test. Every problem is worth 20 points.

1. Suppose that A is the set of sophomores at your school and B the set of students in discrete mathematics at your school. Express each of these sets in terms of A and B.

a. The set of sophomores taking discrete mathematics at your school. **Answer**: $A \cap B$

b. The set of sophomores at your school who are not taking discrete mathematics **Answer**: $A \cap \overline{B}$

c. The set of students at your school who are sophomores or are taking discrete mathematics. **Answer**: $A \cup B$

d. The set of students at your school who are not sophomores or are not taking discrete mathematics. **Answer**: $\overline{A} \cup \overline{B}$

2. The successor of a set A is defined as the set $A \cup \{A\}$.

a. Find the successor of $\emptyset$. Answer: $\emptyset \cup \{\emptyset\} = \{\emptyset\} = 1$

b. Find the successor of $\{\emptyset\}$. Answer: $\{\emptyset\} \cup \{\{\emptyset\}\} = \{\emptyset, \{\emptyset\}\} = \{0, 1\} = 2$

c. Find the successor of $\{\{\emptyset\}\}$ Answer: $\{\{\emptyset\}\} \cup \{\{\{\emptyset\}\}\} = \{\{\emptyset\}, \{\{\emptyset\}\}\} = \{1, \{1\}\}$

d. Find the successor of $\{\emptyset, \{\emptyset\}\}$. Answer: $\{\emptyset, \{\emptyset\}\} \cup \{\{\emptyset, \{\emptyset\}\}\} = \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\} = \{0, 1, 2\} = 3$

3. Mark as true or false.

a. A function f is injective if $f(a) = f(b)$ only if $a = b$. **Answer**: T

b. A function f is injective if $a = b$ then $f(a) = f(b)$. **Answer**: F

c. A function f is injective if $f(a) \neq f(b)$ then $a \neq b$. **Answer**: F

d. A function f is injective if $a \neq b$ then $f(a) \neq f(b)$. **Answer**: T

4. Find an injection $f : \mathbb{N} \twoheadrightarrow \mathbb{N}$ from the set $\mathbb{N}$ of natural numbers to itself which is not surjective. **Answer**: $f : \mathbb{N} \longrightarrow \mathbb{N}, n \mapsto 2n$

5. Determine whether each of these functions f from $\mathbb{Z}$ to $\mathbb{Z}$ is injective. You need to give an explanation.

a. $f(n) = n$. **Answer**: Injective. This is the identity function.

b. $f(n) = n^2$ **Answer**: Not injective, $f(n) = f(-n)$

c. $f(n) = n - 1$ **Answer**: Injective, $n - 1 = m - 1$ yields $n = m$

d. $f(n) = n + n^2$ **Answer**: Not injective. $f(0) = f(-1) == 0$

6. Determine whether each of these functions f from $\mathbb{Z}$ to $\mathbb{Z}$ is surjective.

a. $f(n) = n$. **Answer**: Surjective.This is the identity function.

b. $f(n) = n^2$. **Answer**: Not surjective. $2 \neq n^2$ for every n

c. $f(n) = n - 1$ **Answer**: Surjective. For given m we have $f(m + 1) = m$

d. $f(n) = n + n^2$ **Answer**: Not surjective. $f(n) = n + n^2 = n(1 + n)$ and primes > 2 cannot be in the image.

7. Determine whether each of these statements are true or false.

a. $\emptyset \in \emptyset$ **Answer**: No, there is nothing in the empty set.

b. $\emptyset \in \{\{\emptyset\}\}$ **Answer**: No. The only element of $\{\{\emptyset\}\}$ is $\{\emptyset\}$ which is not the empty set.

c. $\{\emptyset\} = \{\emptyset, \emptyset\}$ **Answer**: Yes. $\emptyset$ is the only element of $\{\emptyset\}$ as well as of $\{\emptyset, \emptyset\}$.

d. $\{\emptyset\} \subseteq \{\{\emptyset, \emptyset\}\}$ **Answer**: No. $\emptyset$ is the only element of $\{\emptyset\}$ which is not an element of $\{\{\emptyset, \emptyset\}\}$.

8. Let $f : A \twoheadrightarrow P(A)$ be a function from A to its powerset. Let $R = \{a | a \notin f(a)\}$. Prove that for no element $s \in A$ one can have $f(s) = R$.
Answer: Assume for sone s that $f(s) = \{a | a \notin f(a)$. We then would have $s \in R$ iff $s \notin f(s) = R$, which is not possible.

9 Is it the case that an injective function $f : S \longrightarrow S$ from a finite set to itself is always surjective? You must explain your answer.
Answer: Yes. If S is a finite set with $n - many$ elements, then because of injectivity, there are n –many images. Thus every element of S is an image. Thus f is urjective.

10. You must prove your answers for

a Is the set $\emptyset$ the power set of a set?**Answer**: No. any powerset contains at least one element, namely the emptyset.

b. b)Is $\{\emptyset, \{\{\emptyset\}\}\}$, the power set of a set? **Answer**: Yes. $P(\{\emptyset\}) = \{\emptyset, \{\emptyset\}\}$